Parameterized Analysis of Immediate Observation Petri Nets

Chana Weil-Kennedy

joint work with Javier Esparza and Mikhail Raskin

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European Research Council

Established by the European Commission

Introduction

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\end{array}
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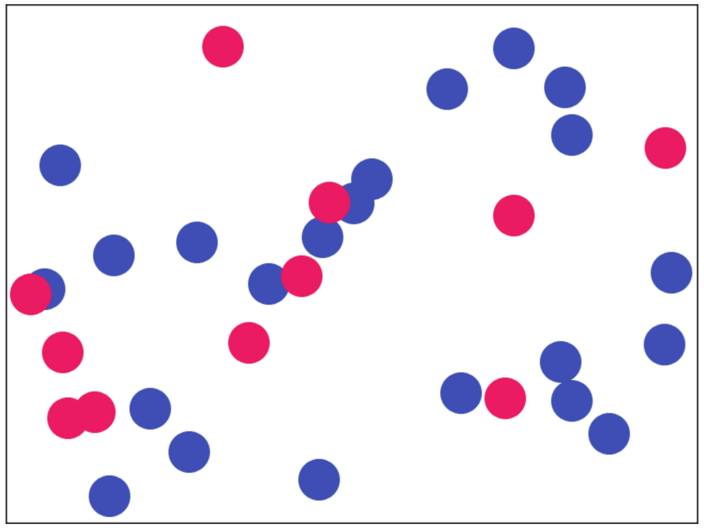

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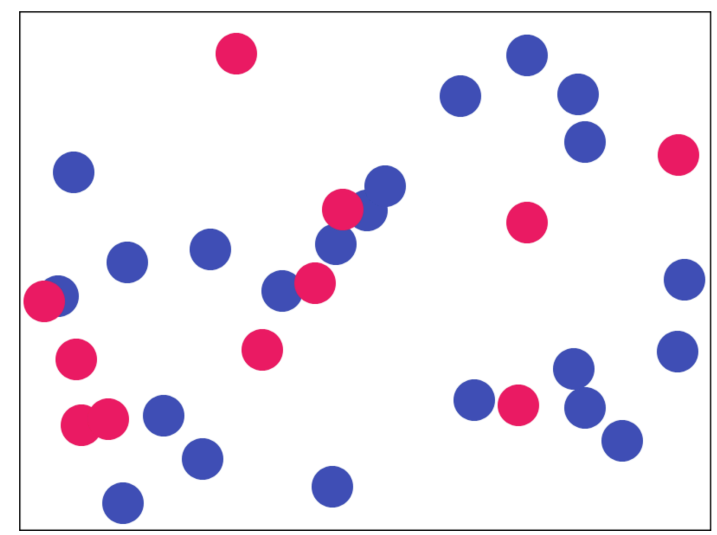
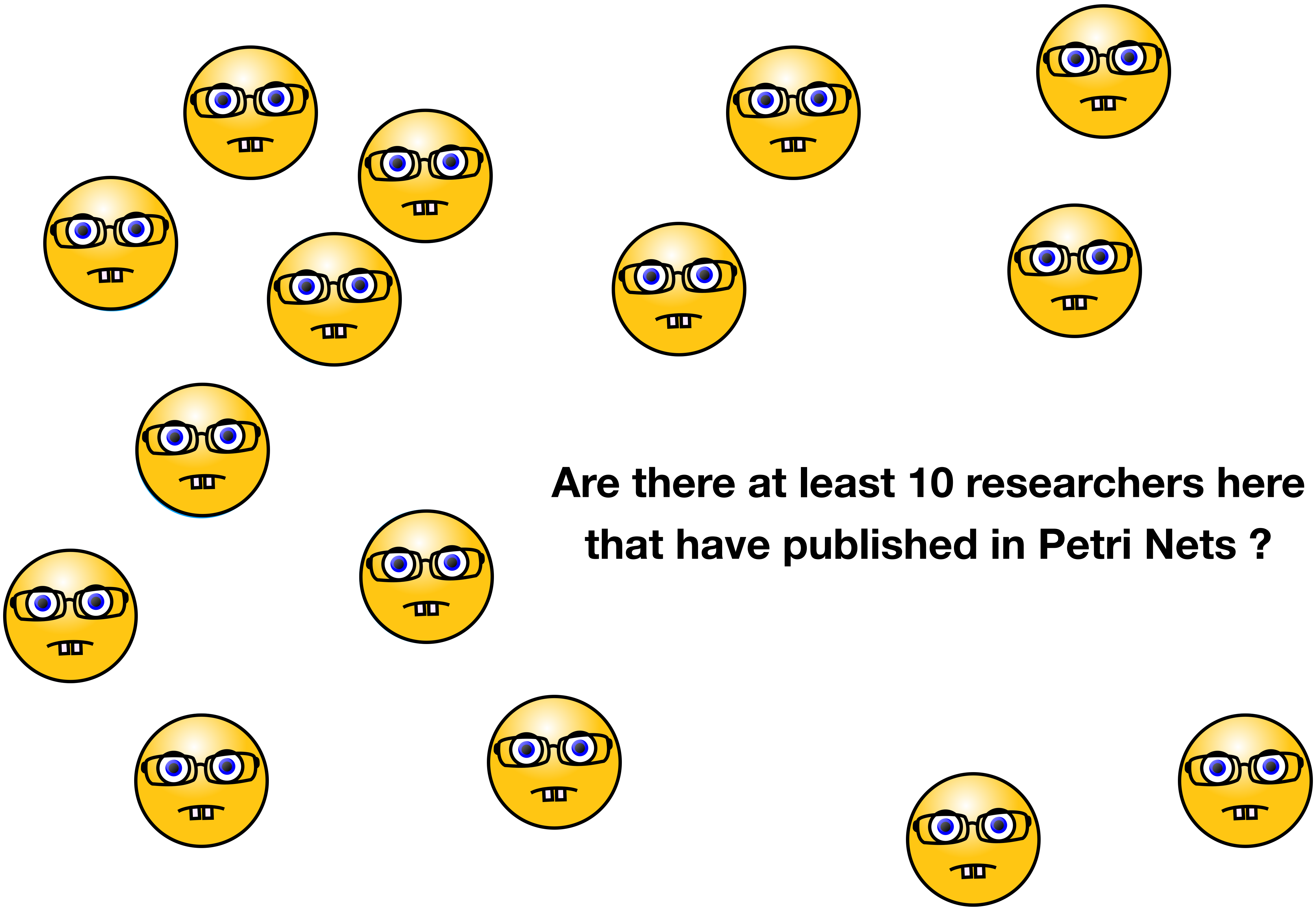


- We study **parameterized** problems for this class.

Researchers in a Room



Are there at least 10 researchers here that have published in Petri Nets ?

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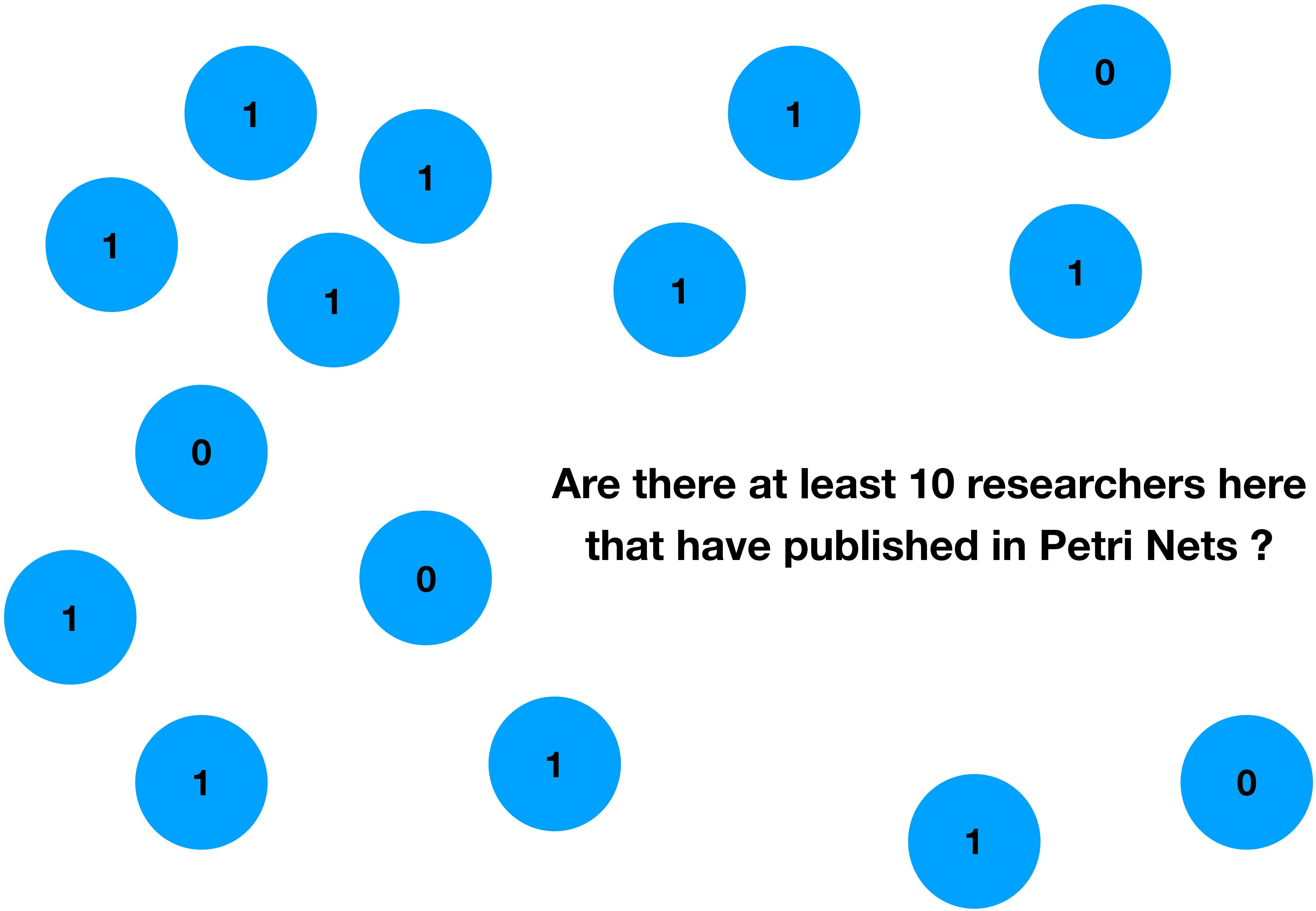

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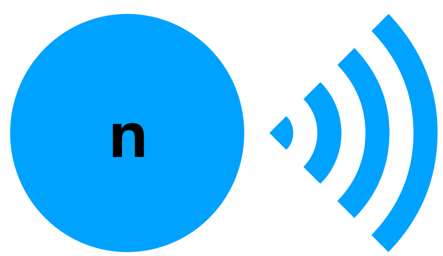

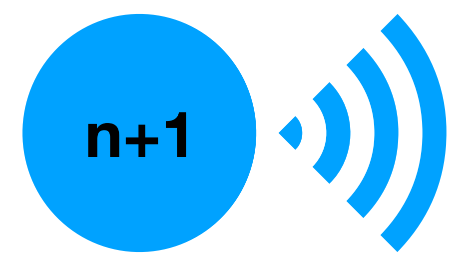

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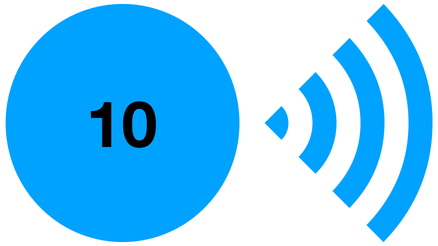
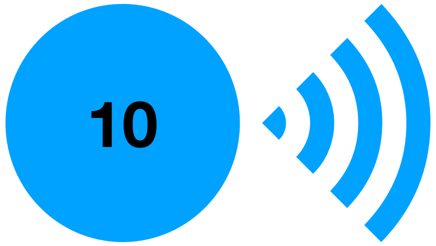
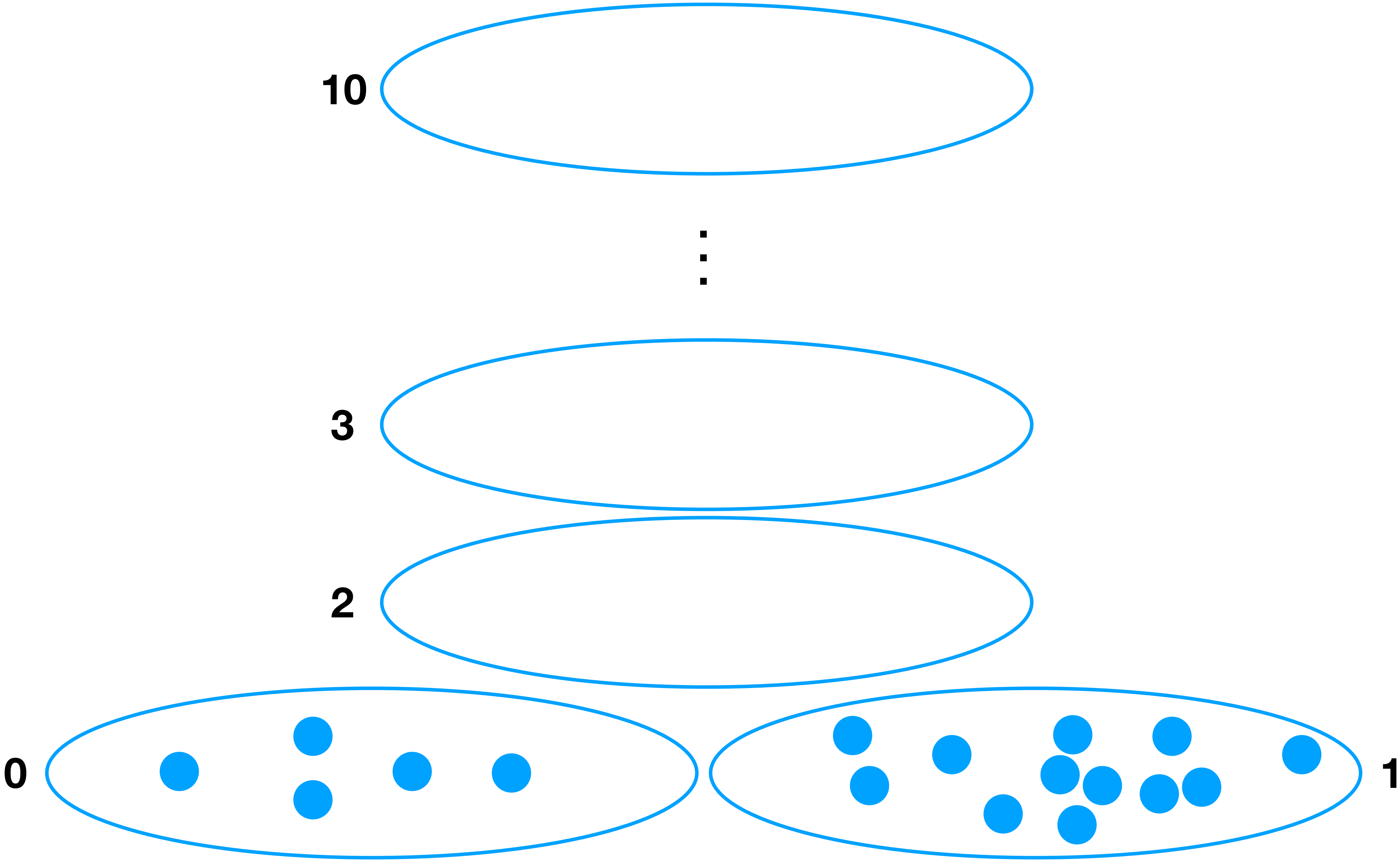




Counting Researchers Protocol



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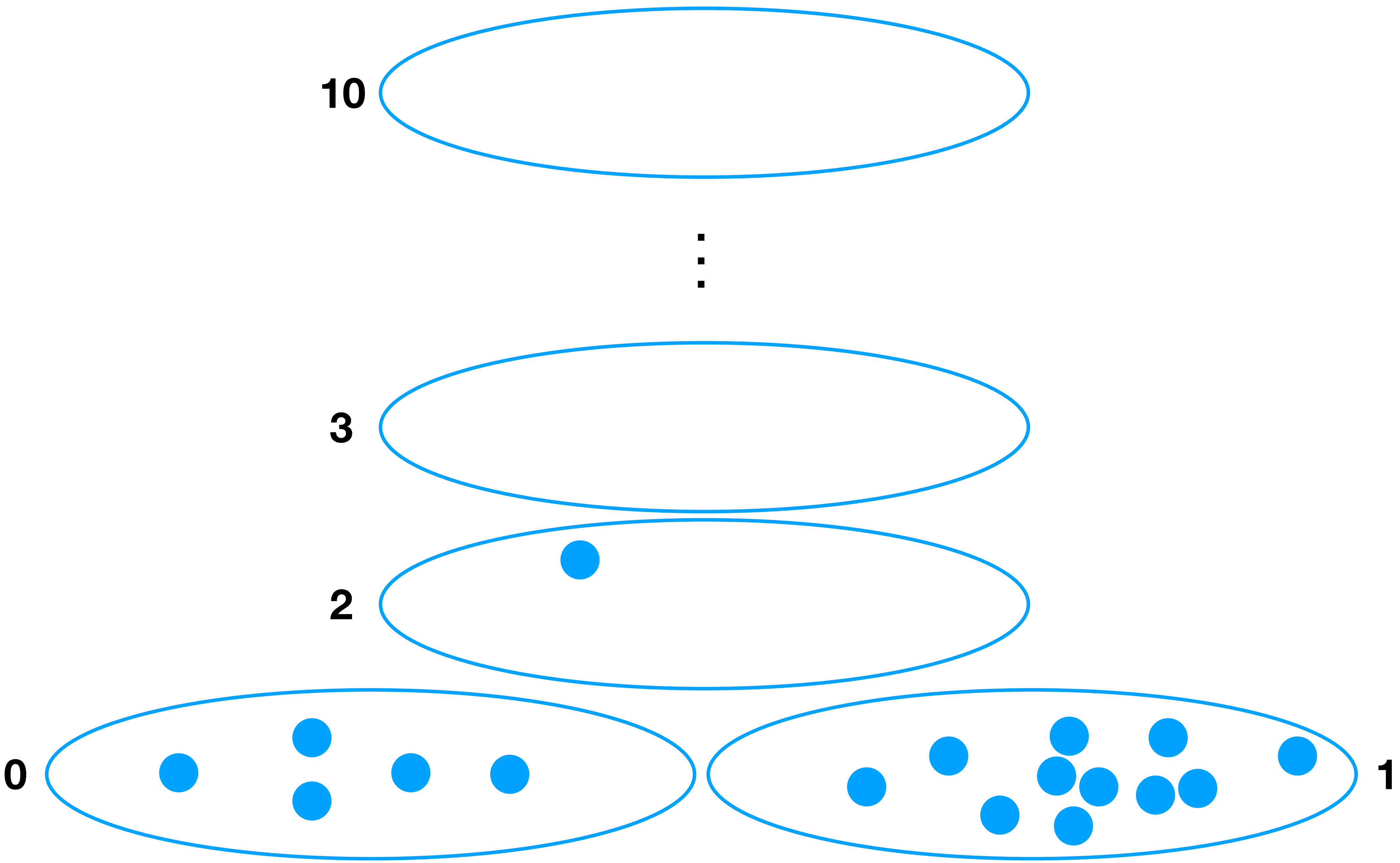

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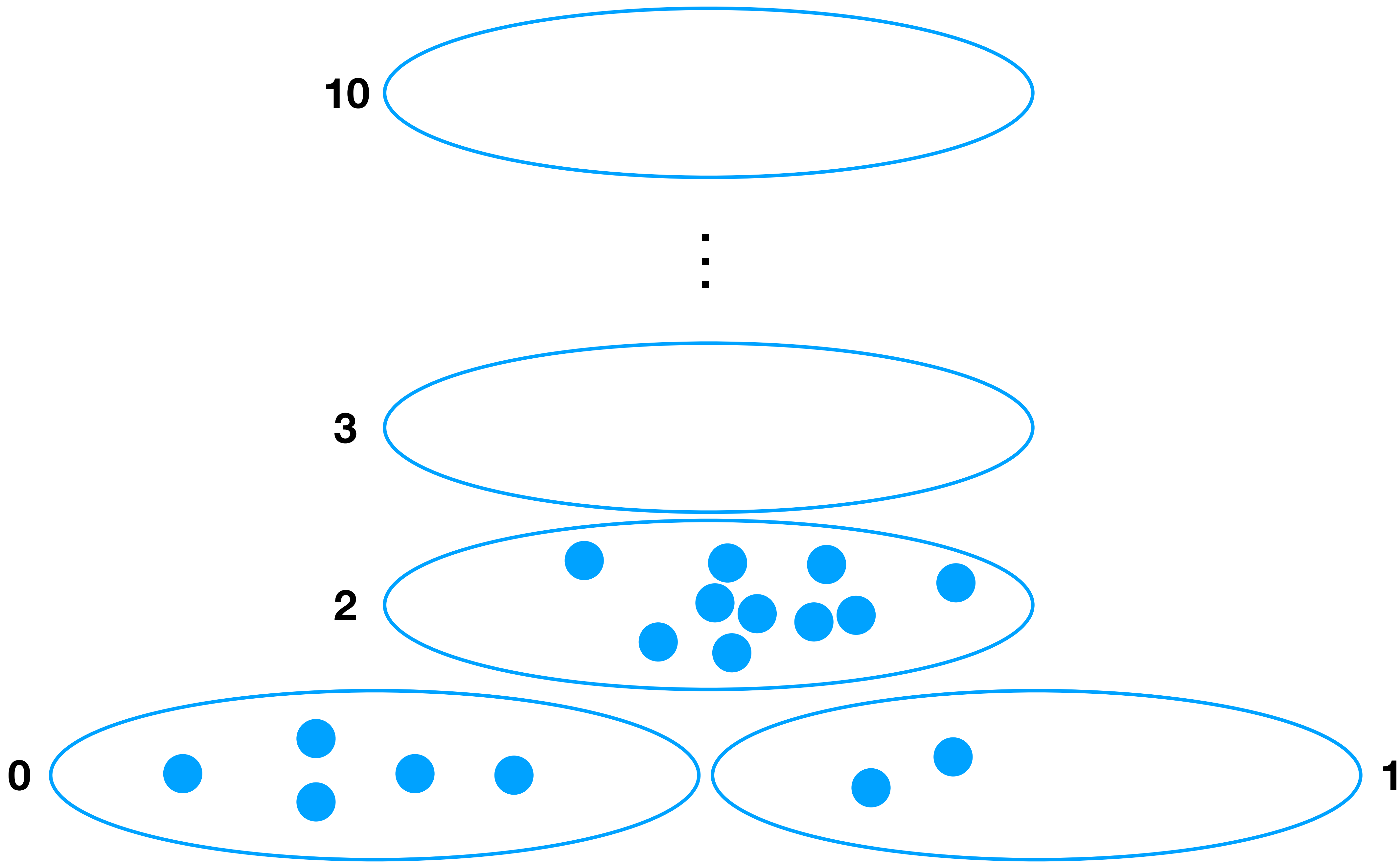

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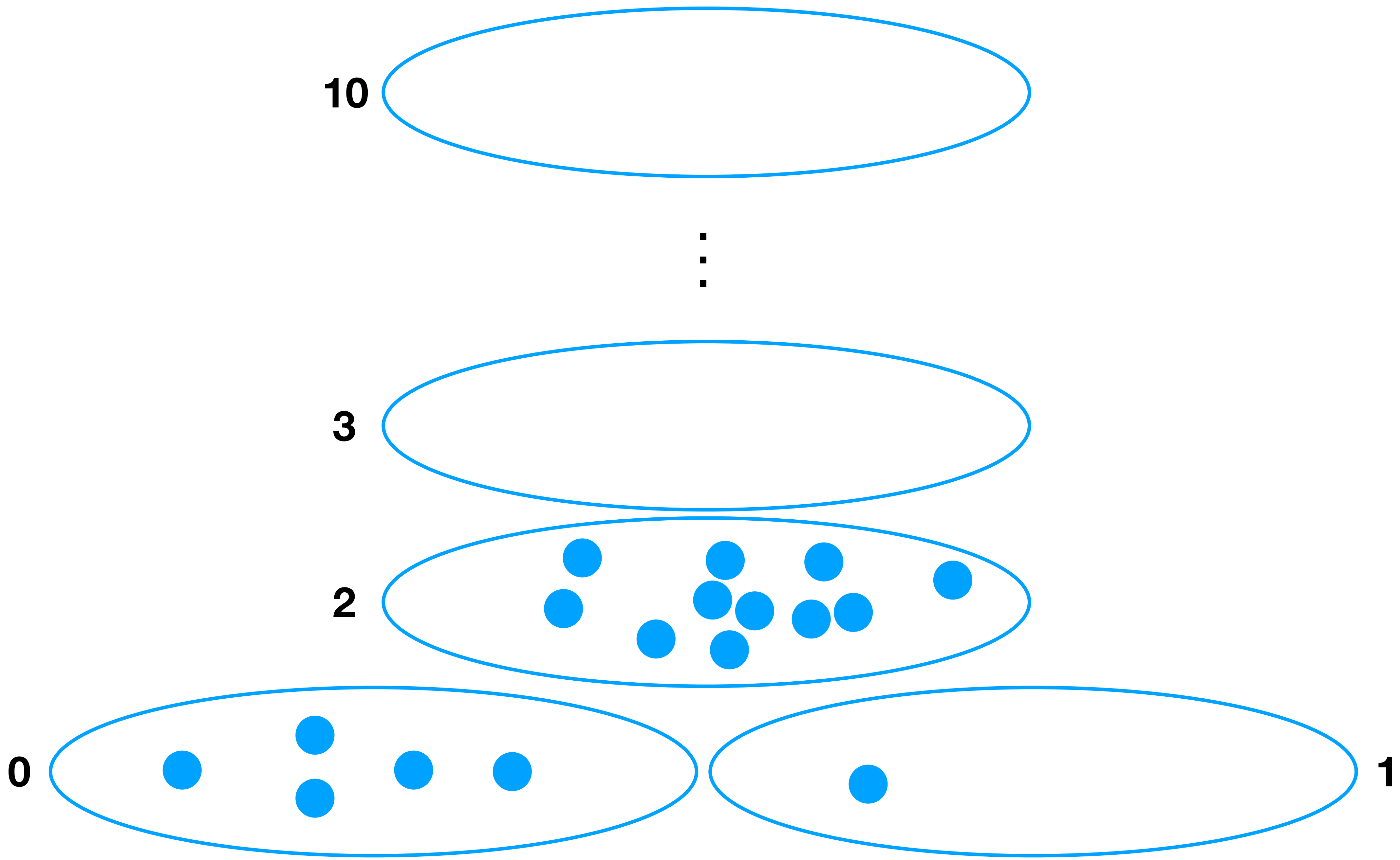

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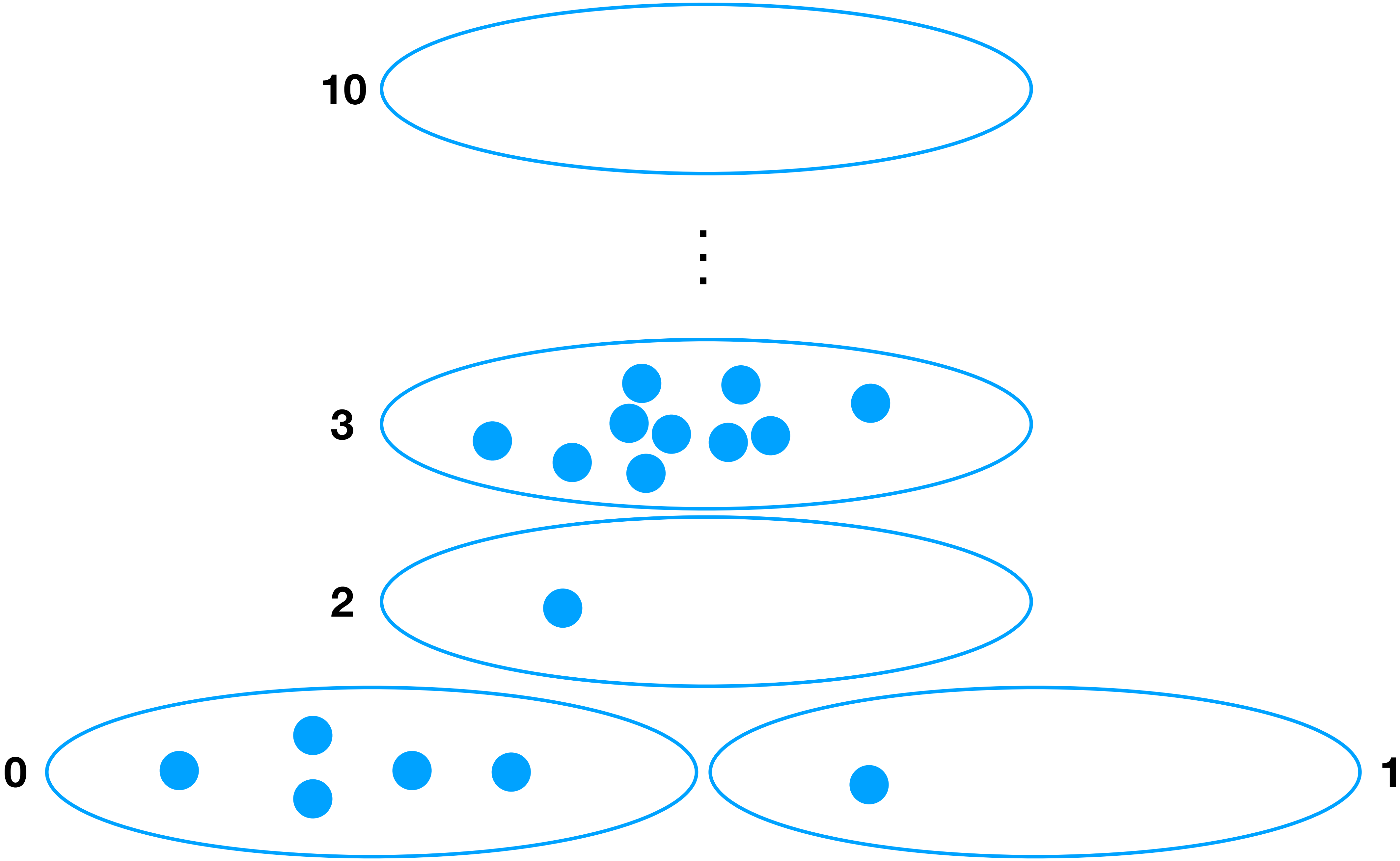

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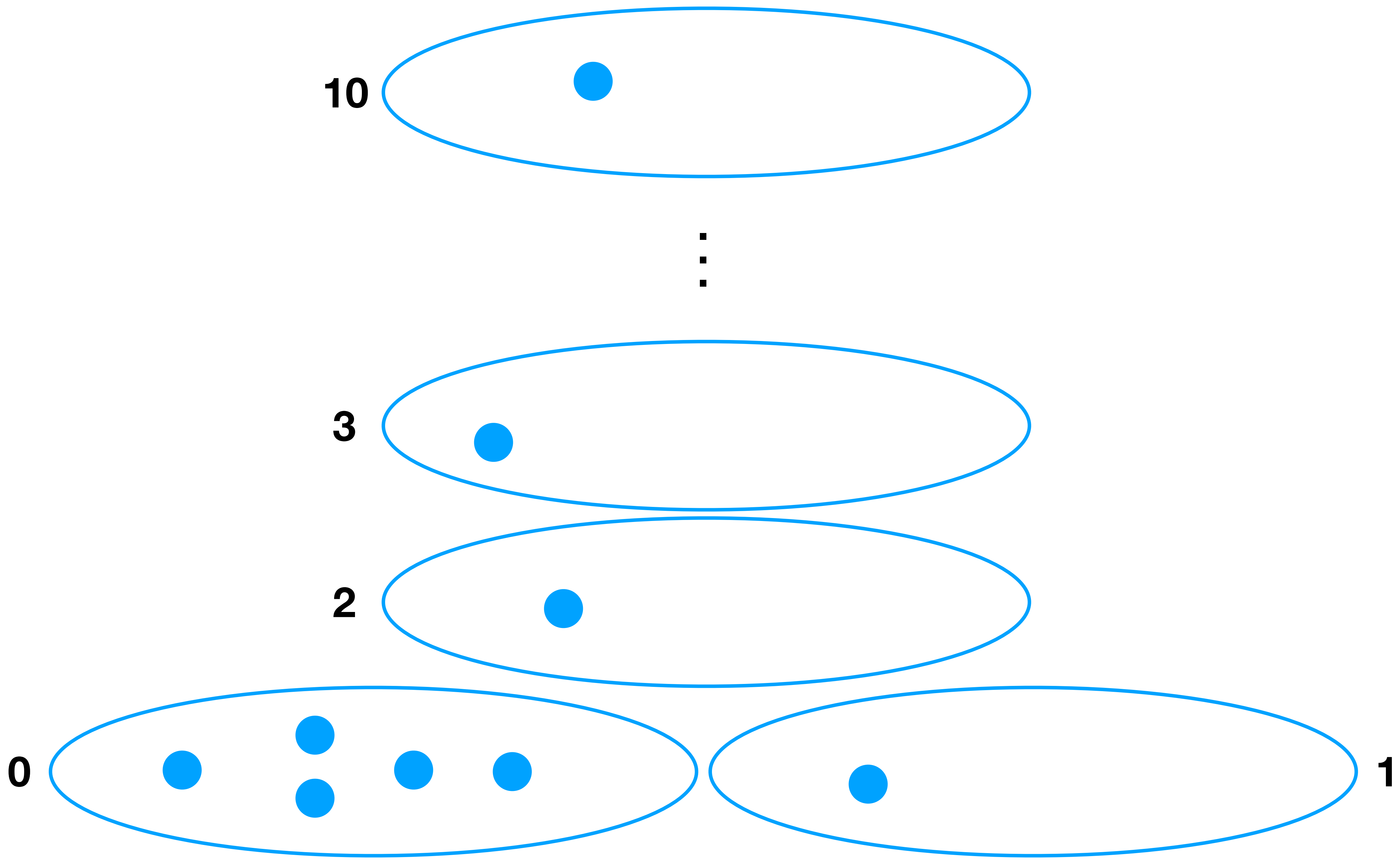

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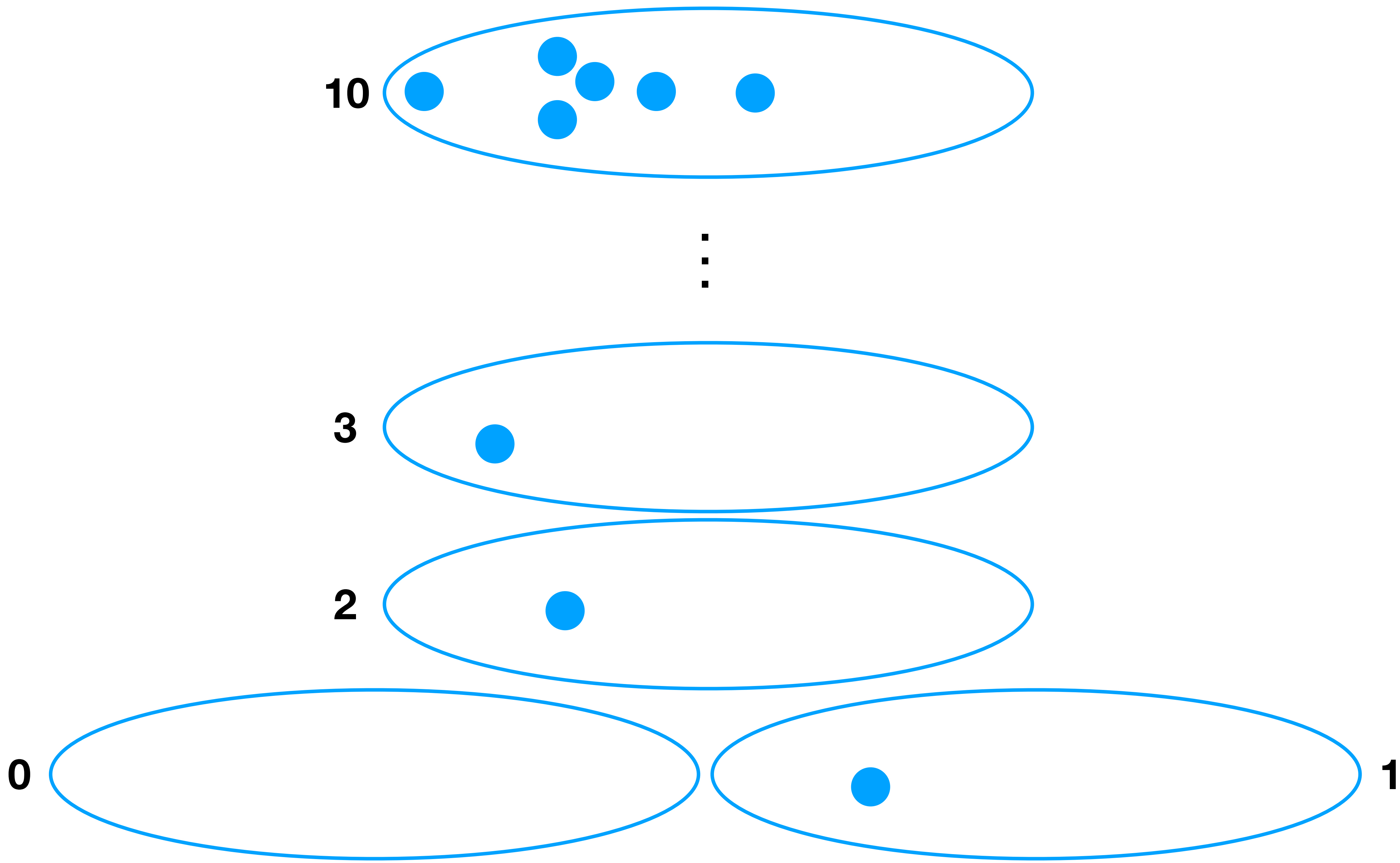

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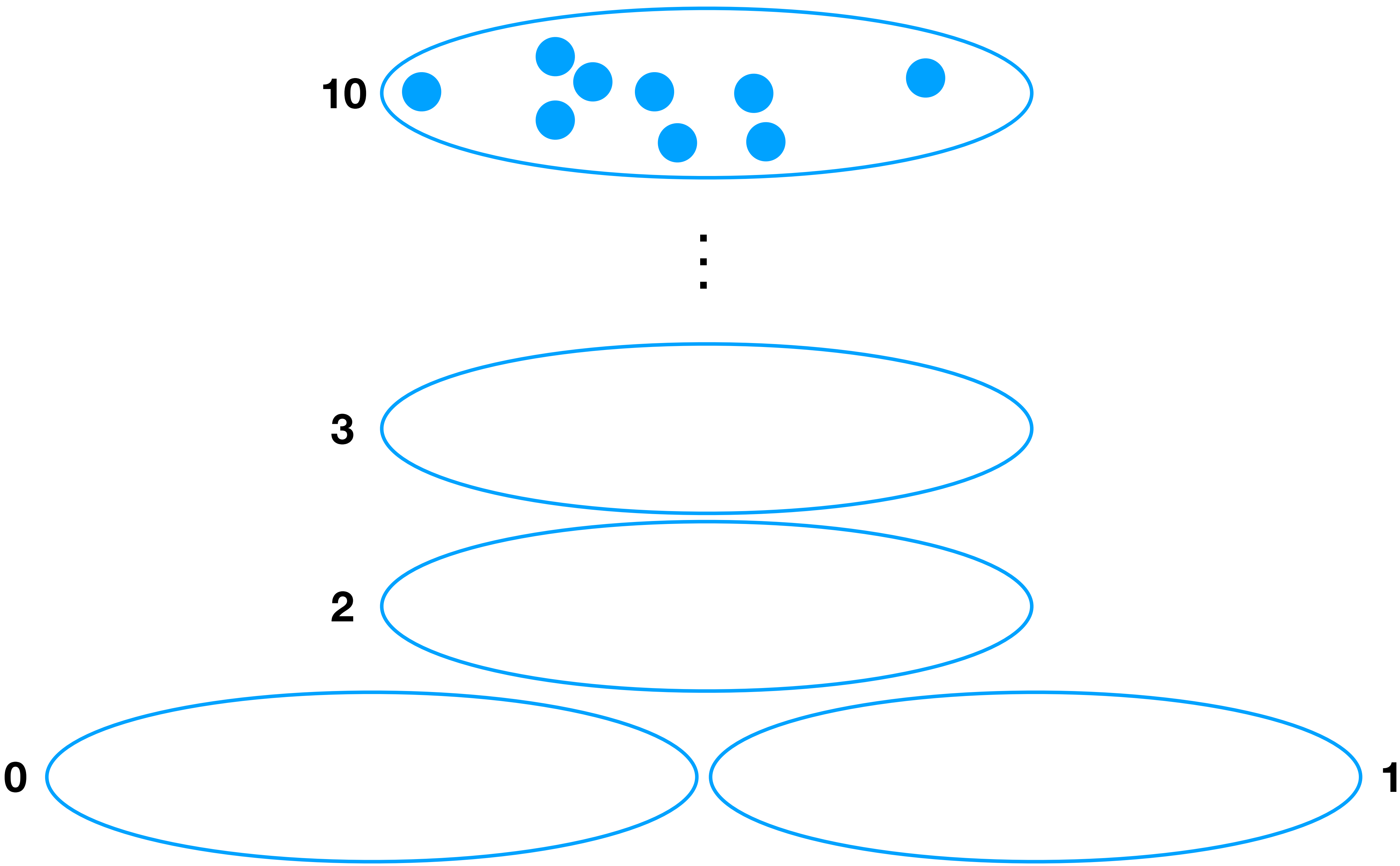
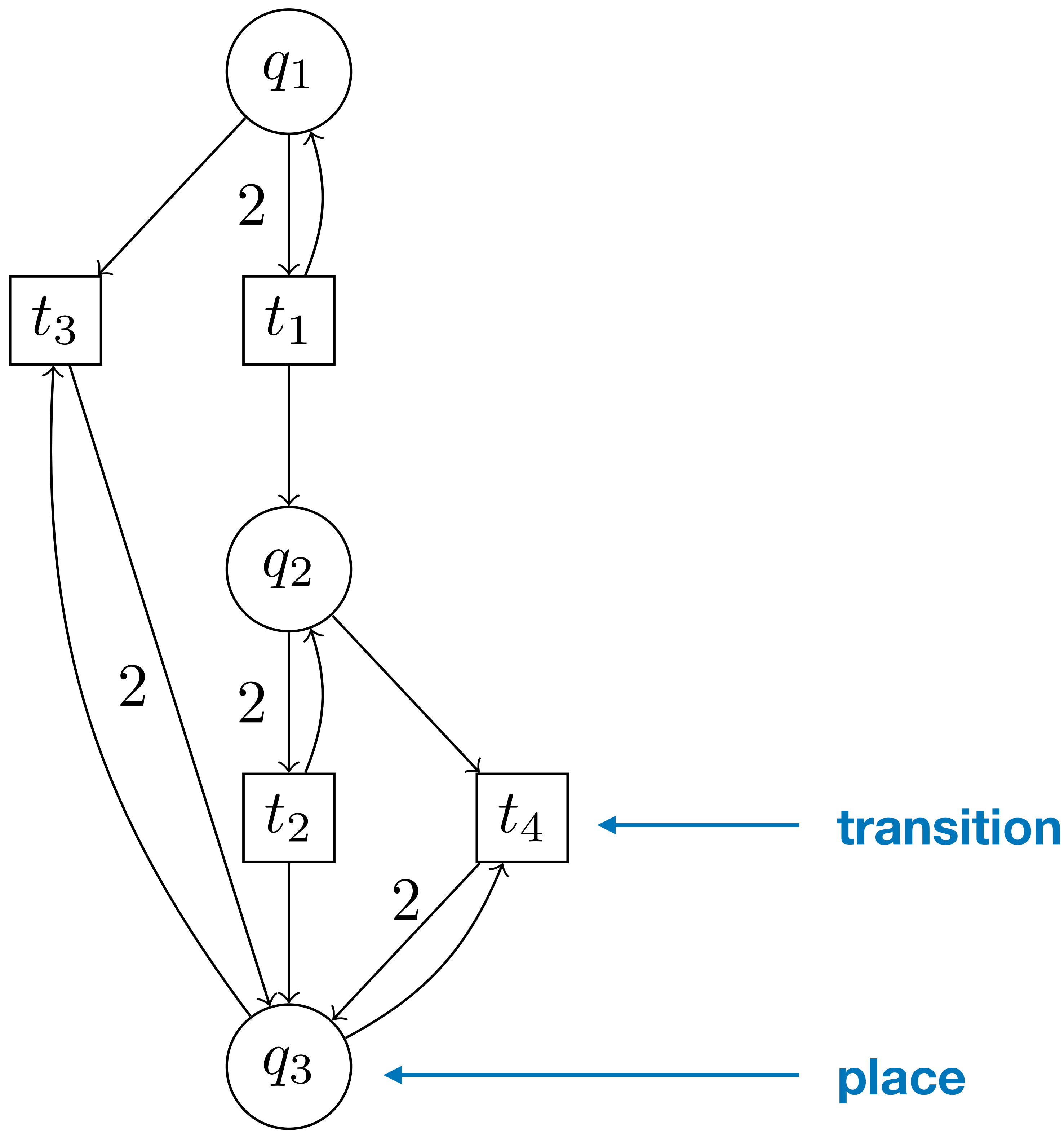


Immediate Observation Nets



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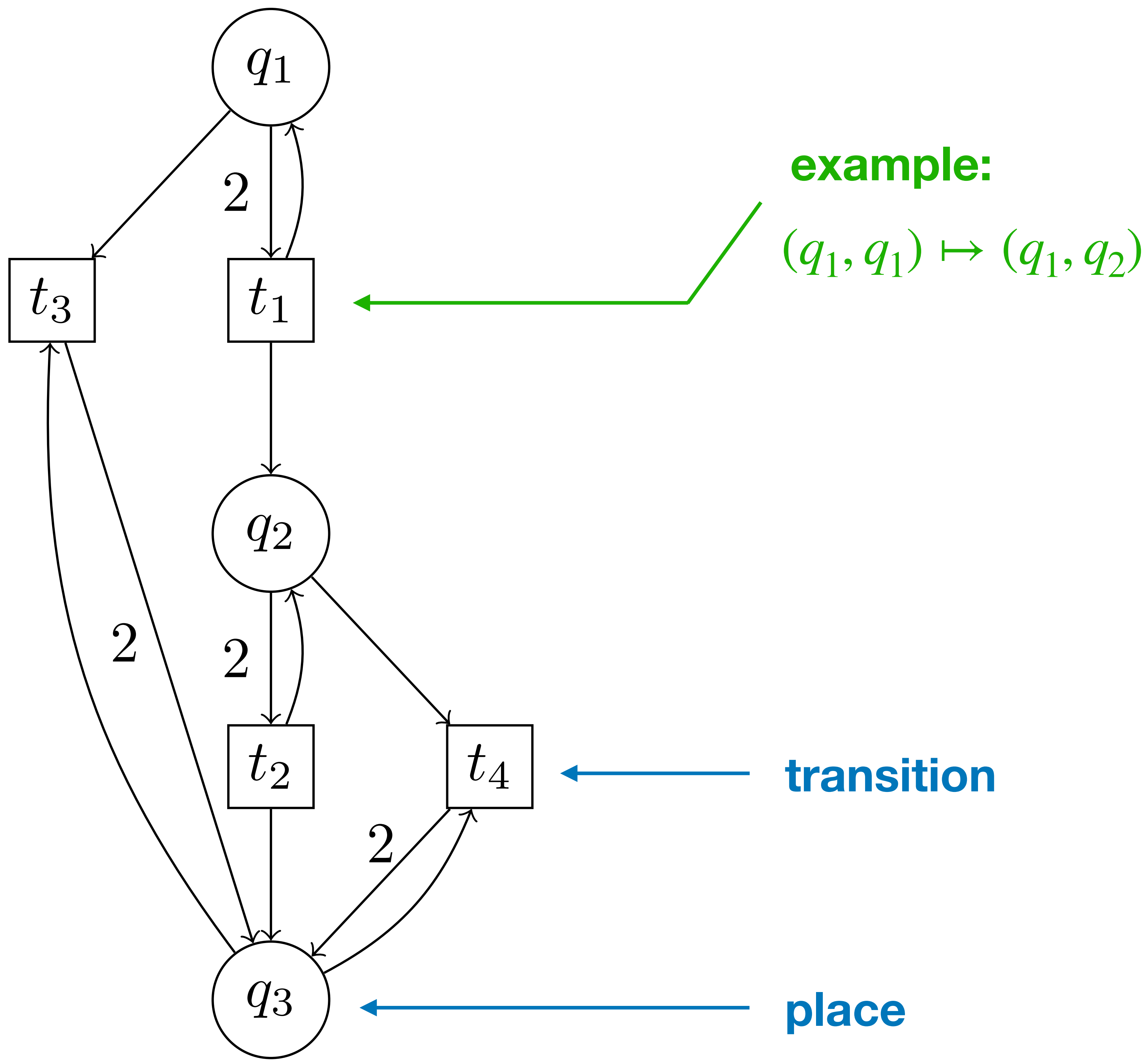

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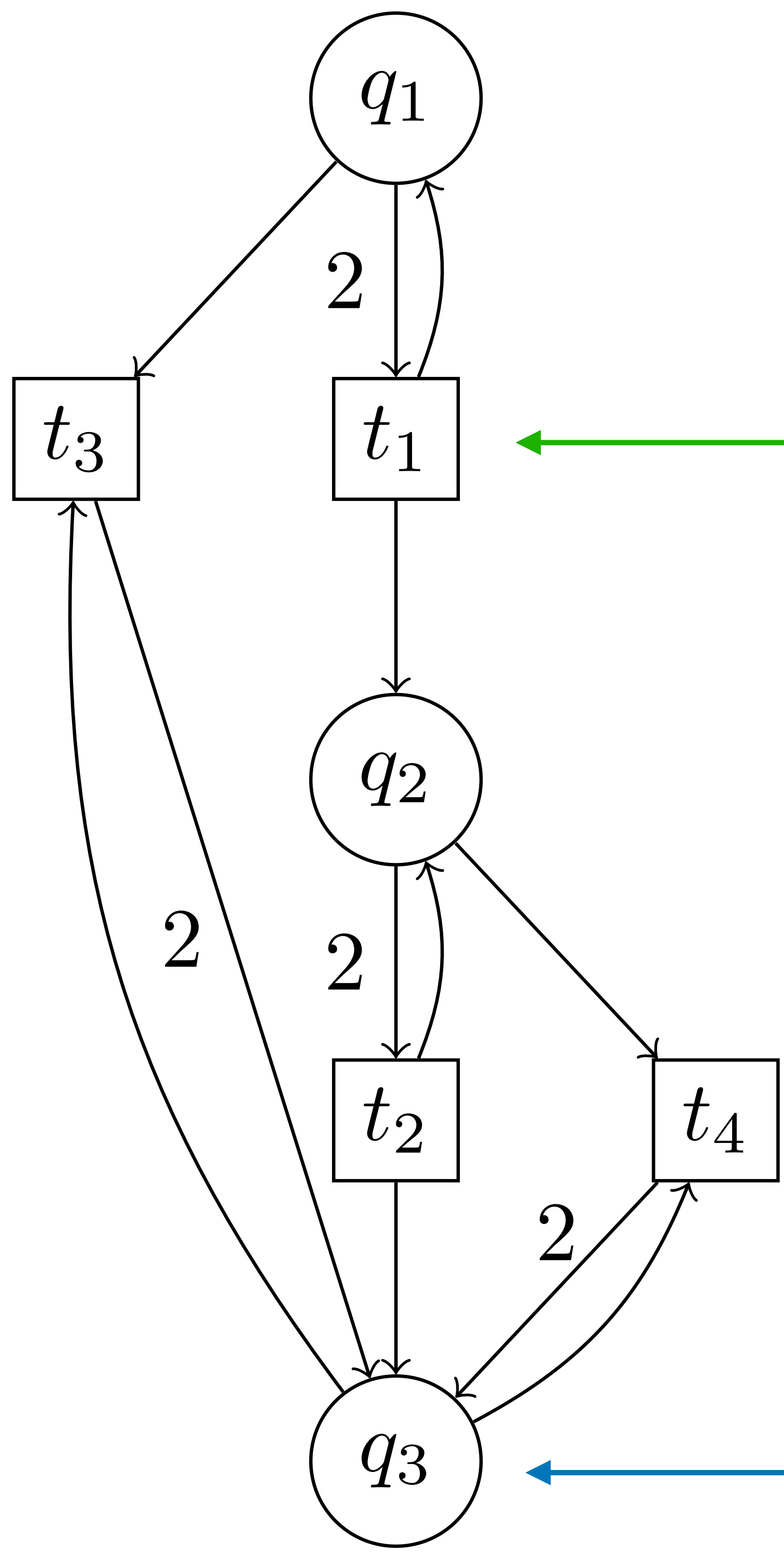

example:

$(q_1, q_1) \mapsto (q_1, q_2)$

A Petri net is an **IO net** if all its transitions are of the form $(q_s\ ,\ q_o) \mapsto (q_d\ ,\ q_o)$

transition

place

Parameterized Problems

Parameterized problems in the number of agents (population protocols) or molecules (enzymatic chemical networks)

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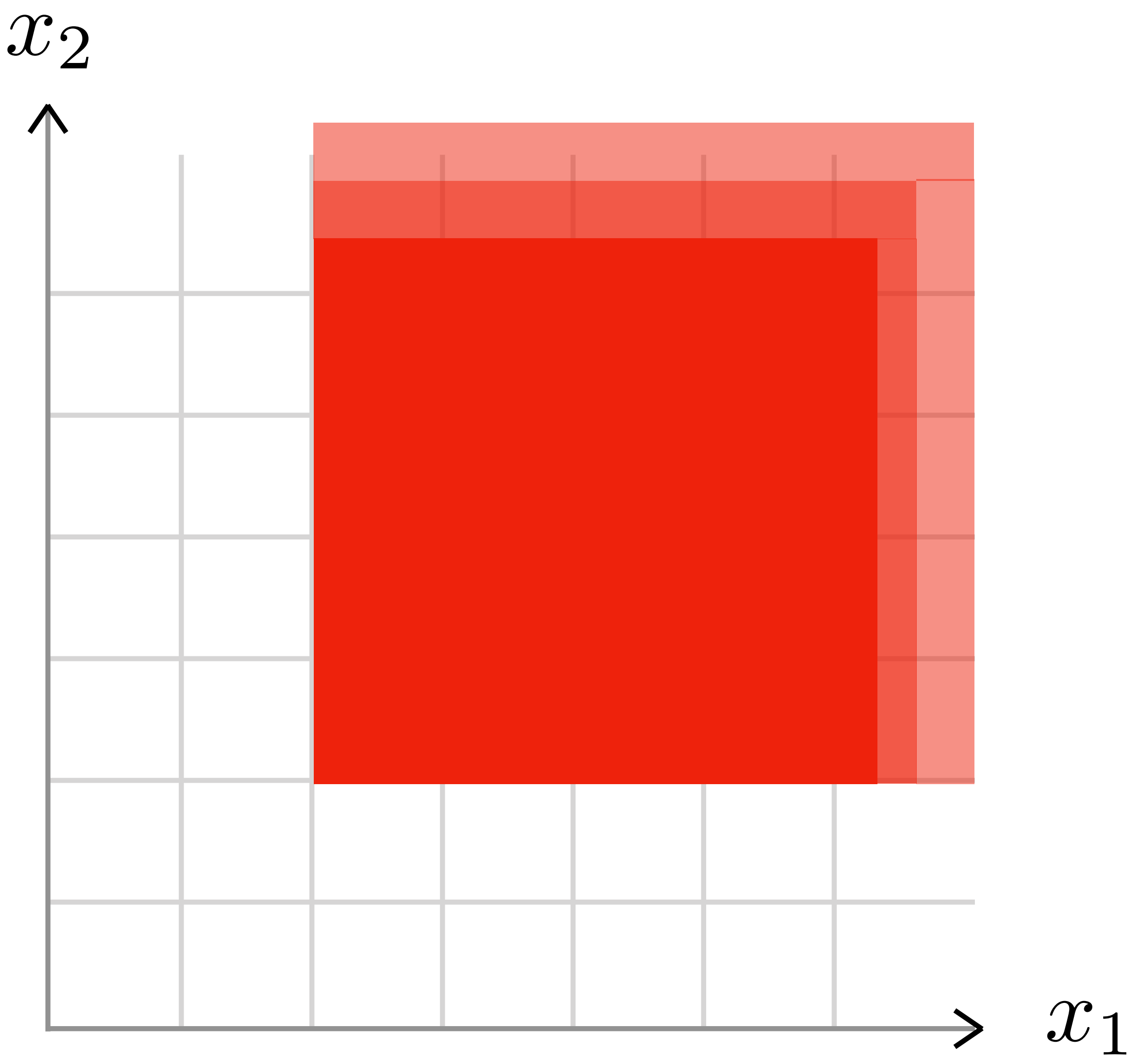
defined by **counting constraints**

Counting Constraints



$$2 \leq x_1 \leq \infty$$
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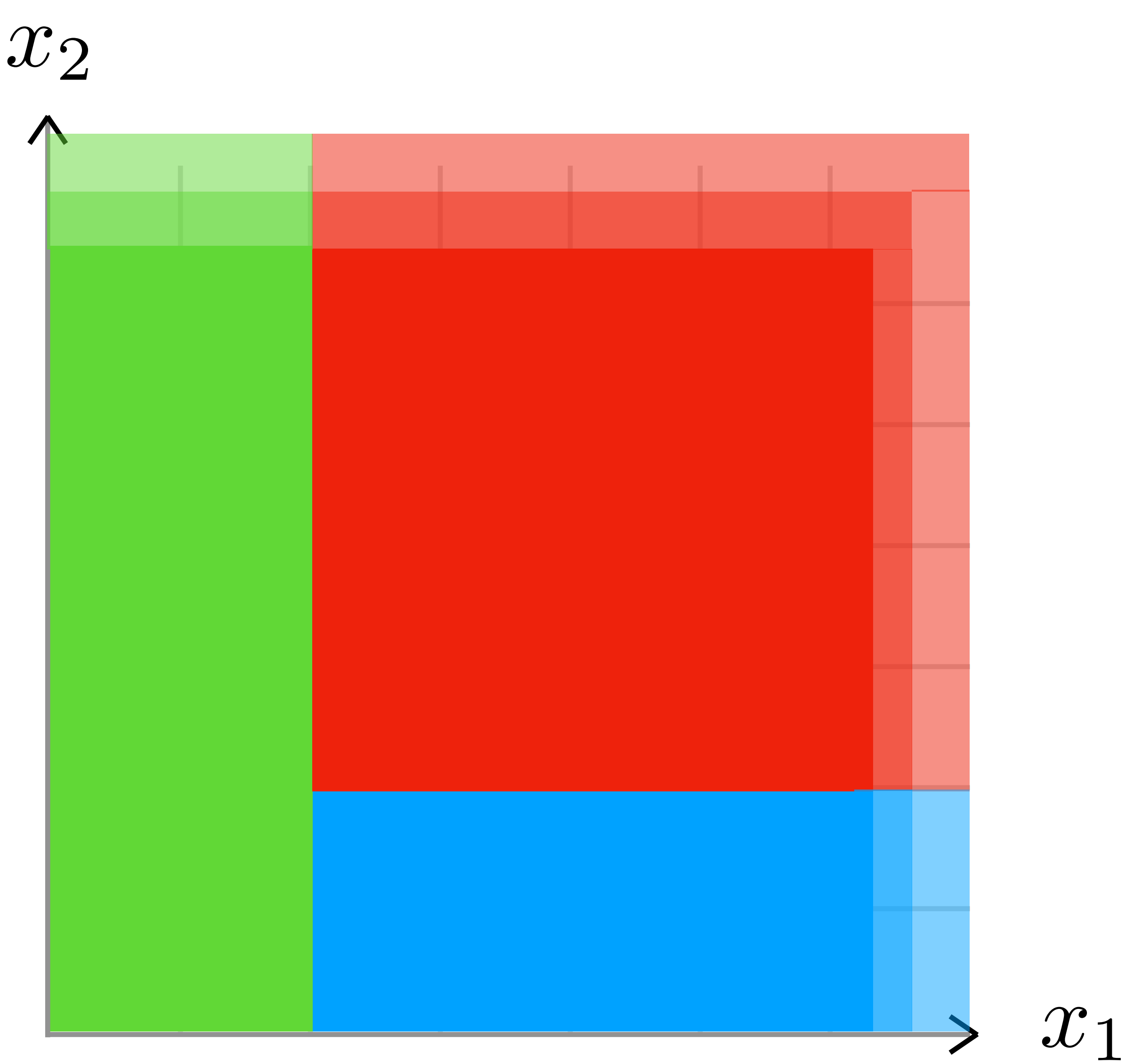
Counting Constraints



$$\neg \begin{array}{l} 2 \leq x_1 \leq \infty \\ 2 \leq x_2 \leq \infty \end{array}$$

$$\Downarrow$$

$$\begin{array}{l} 2 \leq x_1 \leq \infty \\ 0 \leq x_2 \leq 2 \end{array} \quad \vee \quad \begin{array}{l} 0 \leq x_1 \leq 2 \\ 0 \leq x_2 \leq \infty \end{array}$$

Results

Reachability, coverability and liveness are

PSPACE-complete

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Application: the correctness problem for IO population protocols is **PSPACE-complete**

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Express our problems as **formulas over counting constraints** using boolean operators and reachability operators

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using the **Pruning Theorem**

The Pruning Theorem

For any run

$$M \xrightarrow{*} M' \geq M''$$

$$\geq \qquad \geq$$

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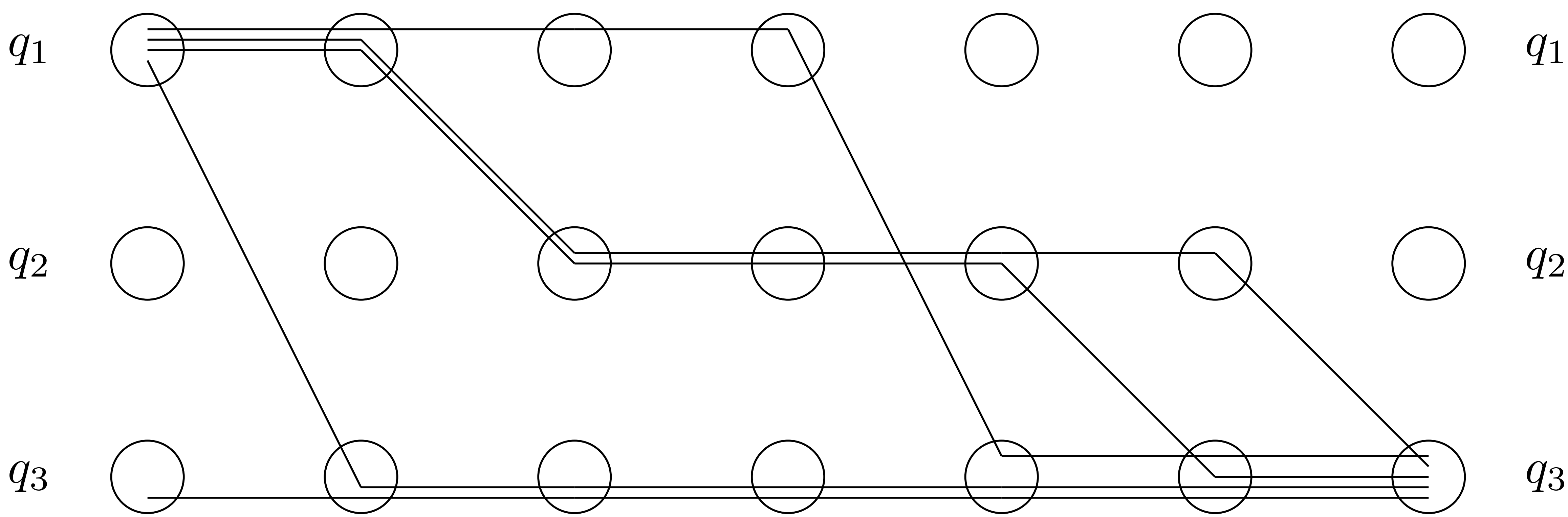

History = the trajectories of the tokens of a run

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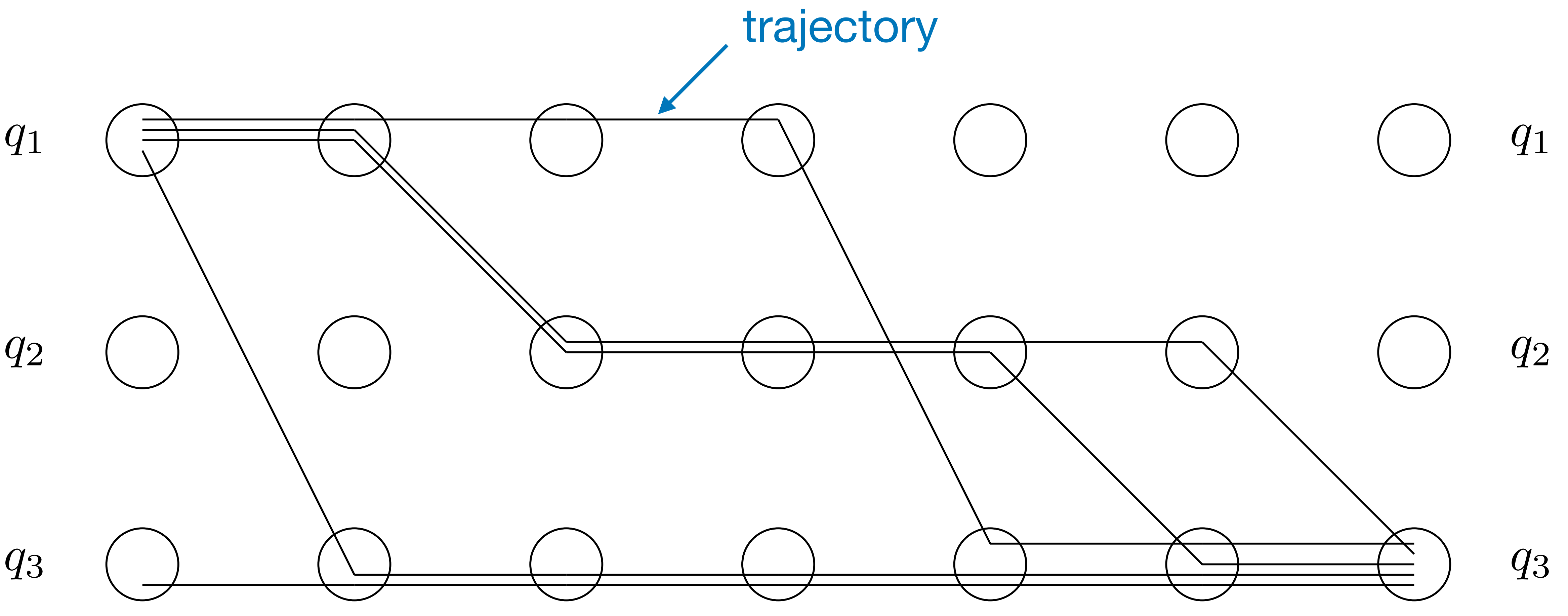

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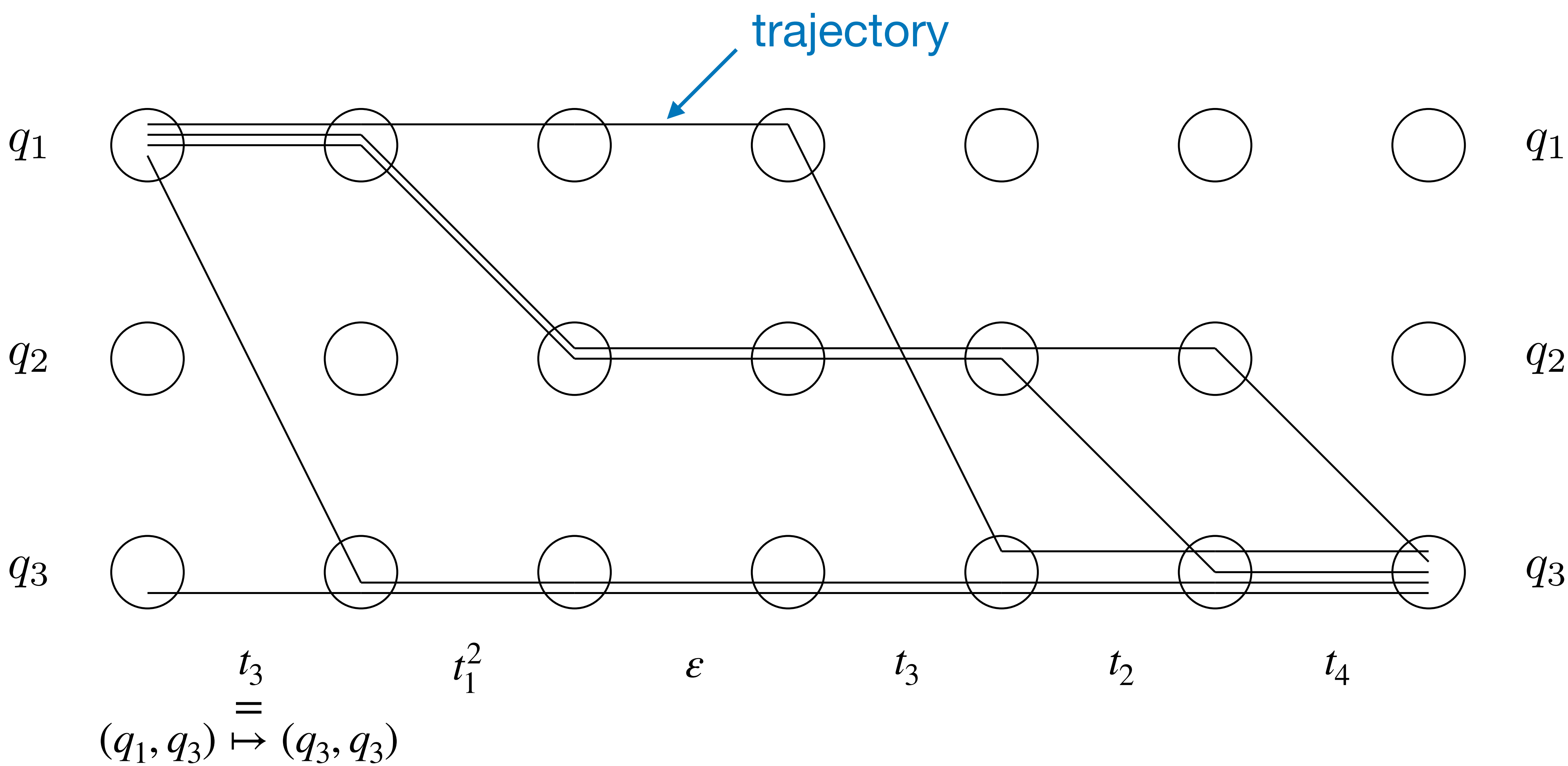

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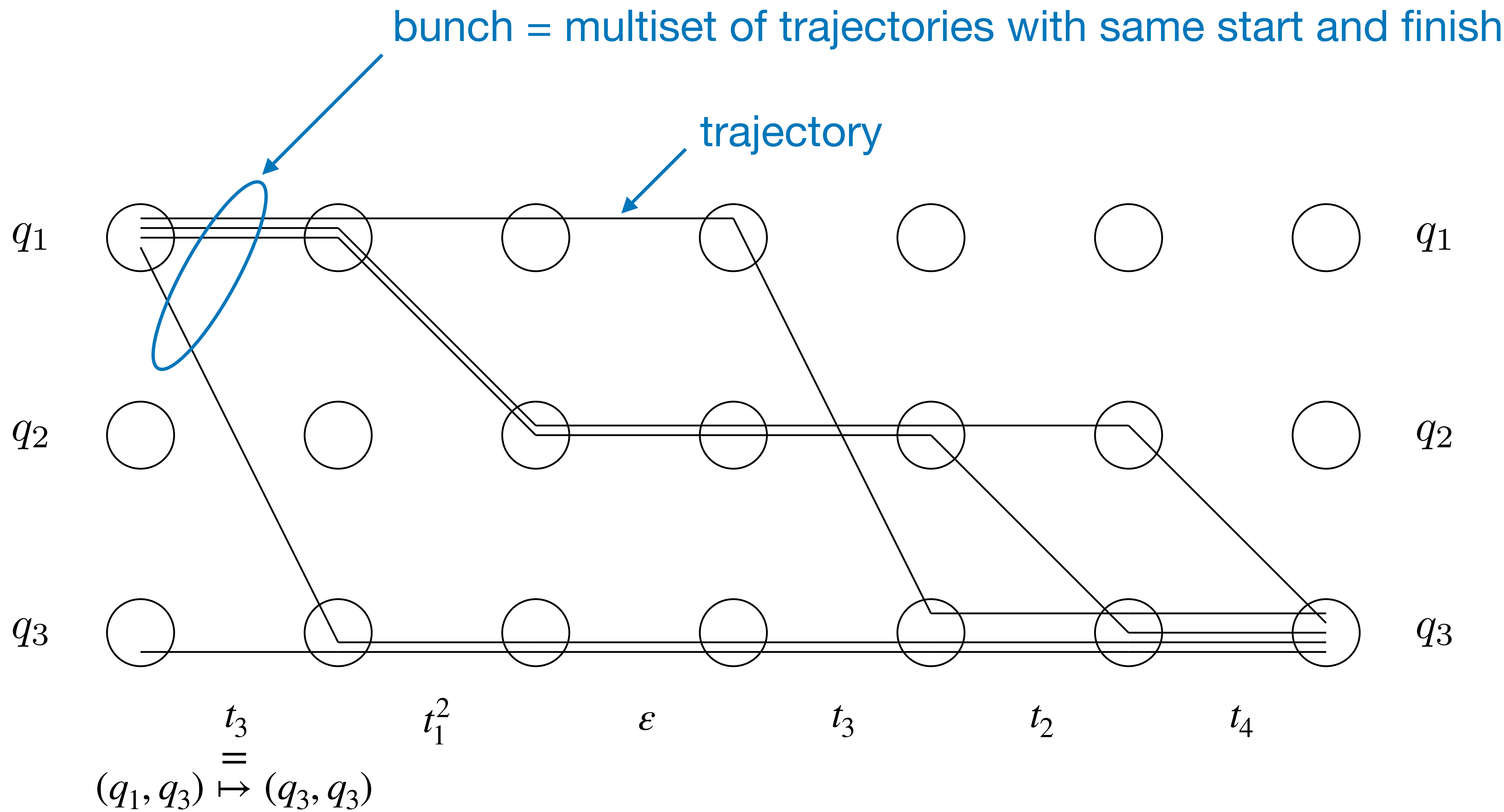

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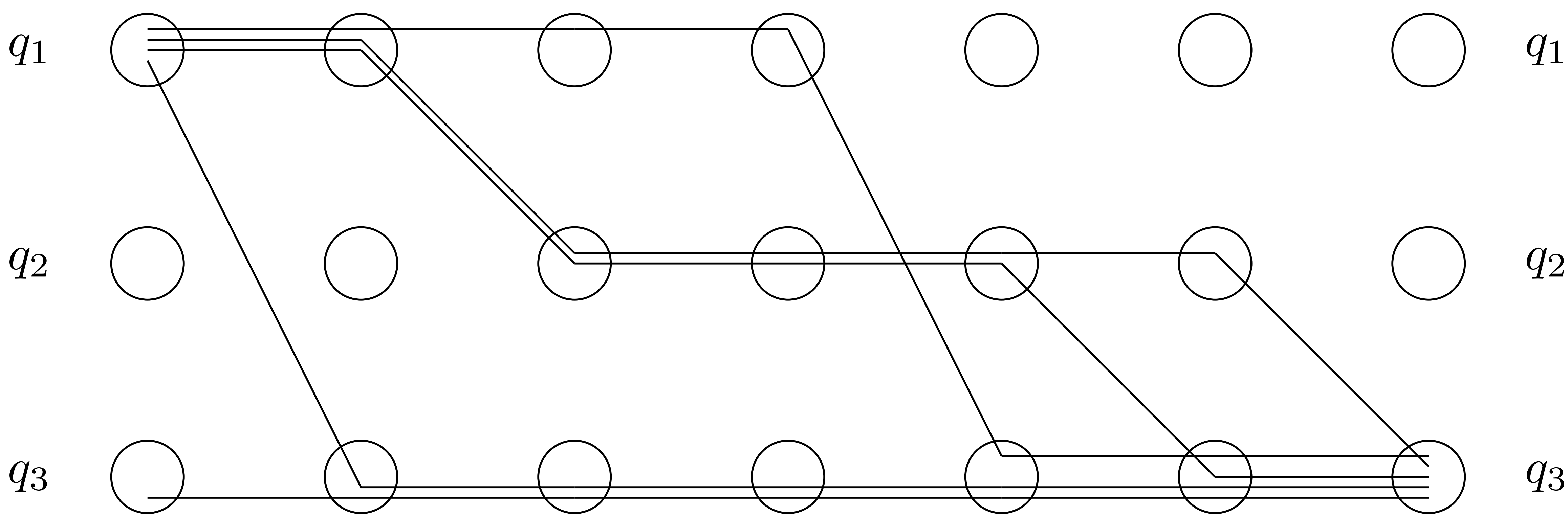
An **IO transition** is

$$(q_s\ ,\ q_o) \mapsto (q_d\ ,\ q_o)$$



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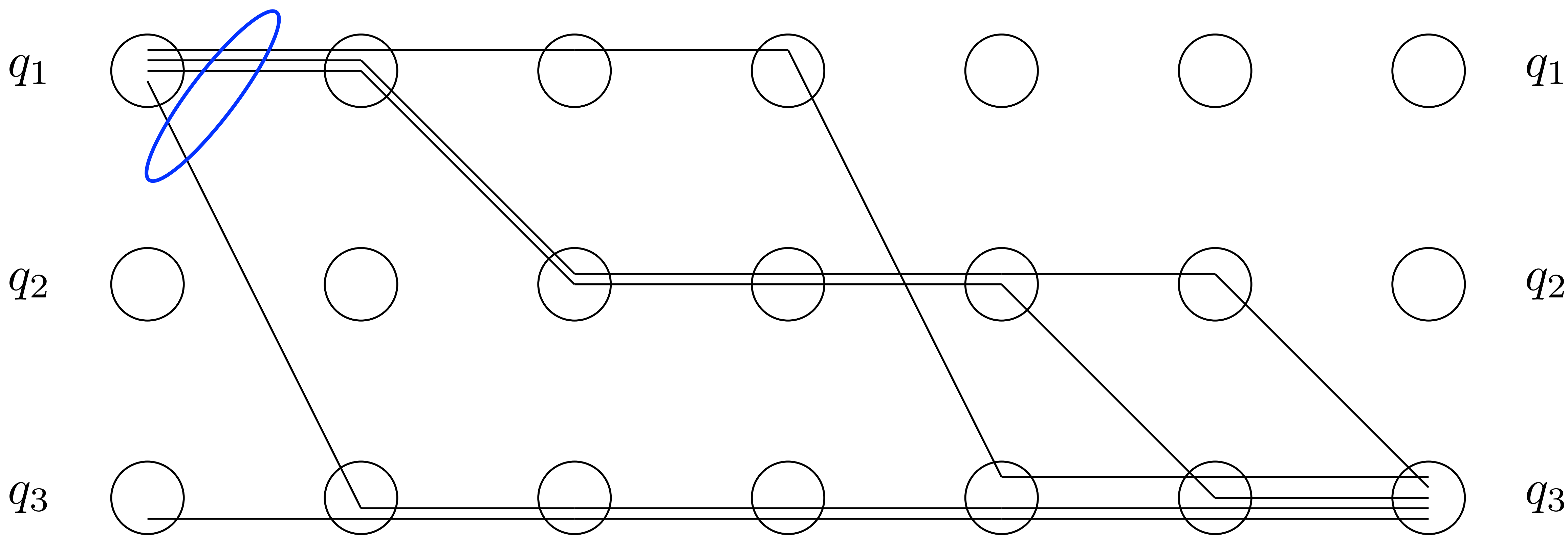

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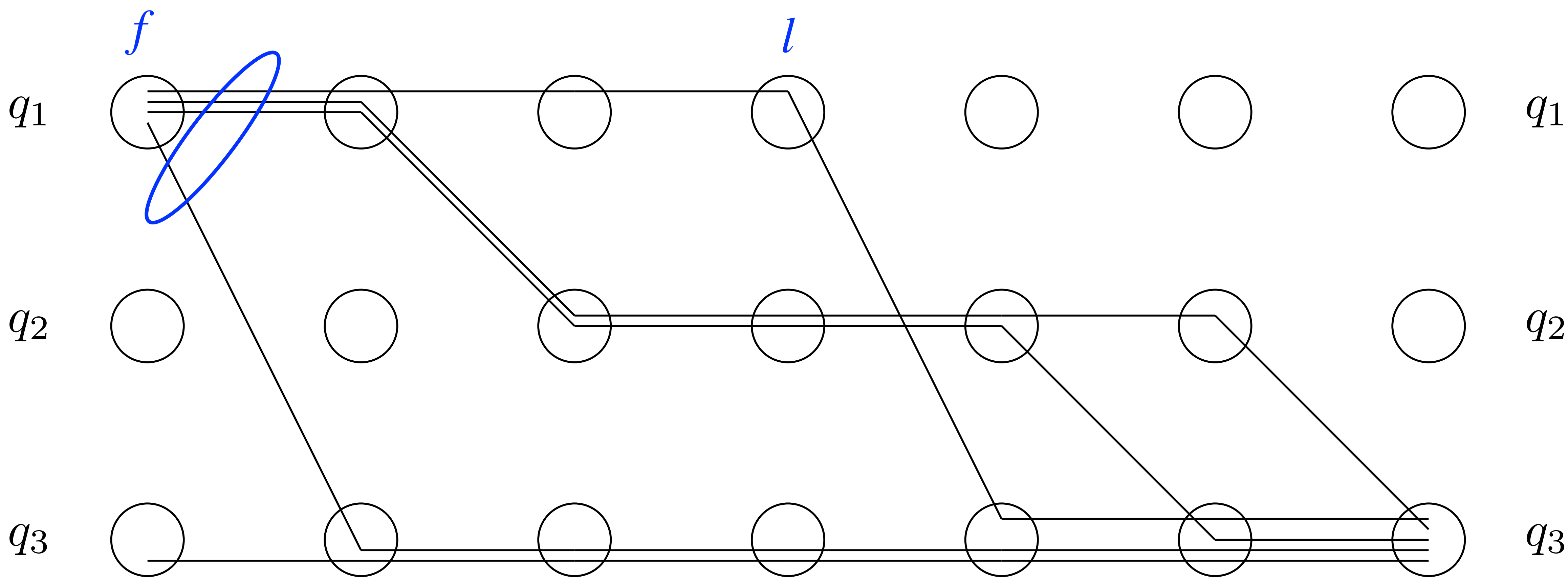

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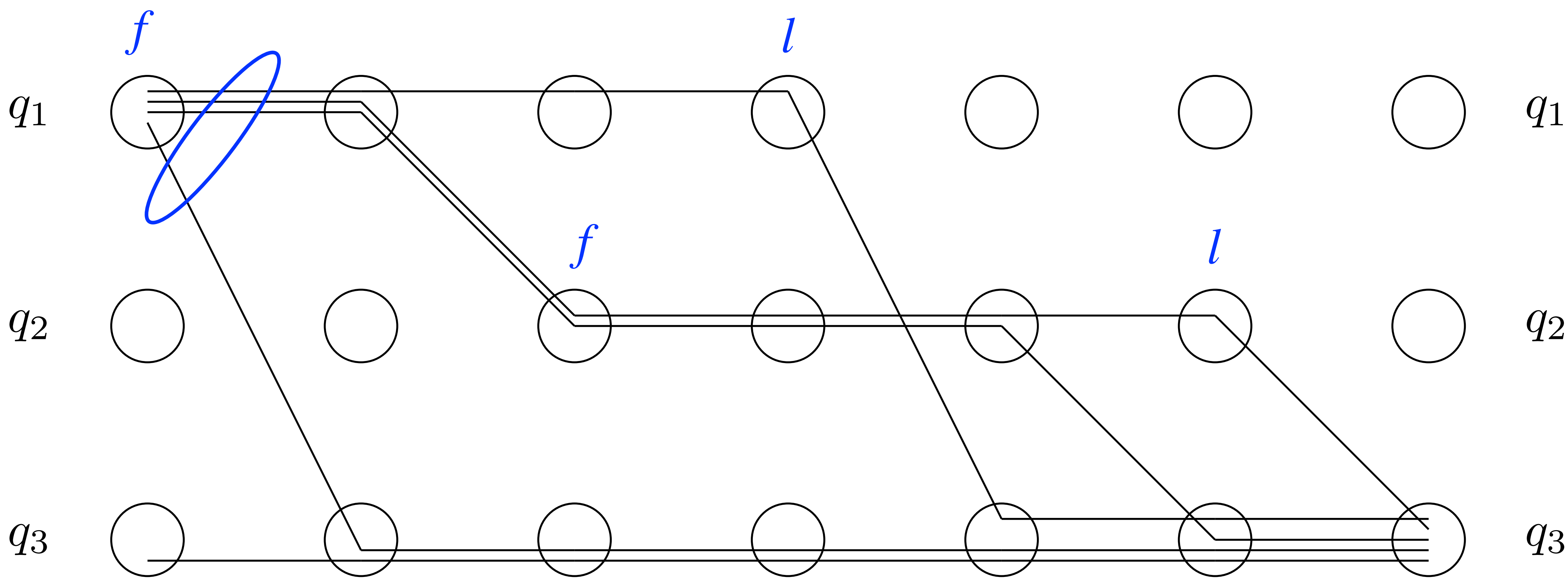

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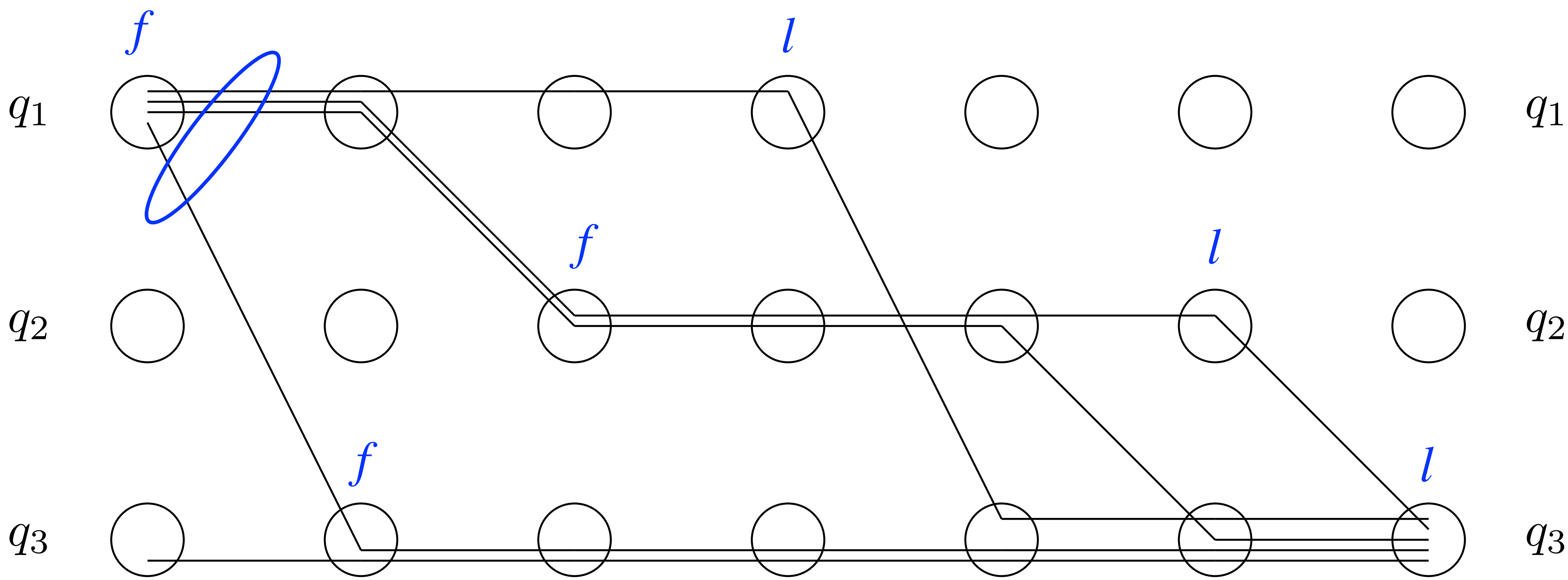

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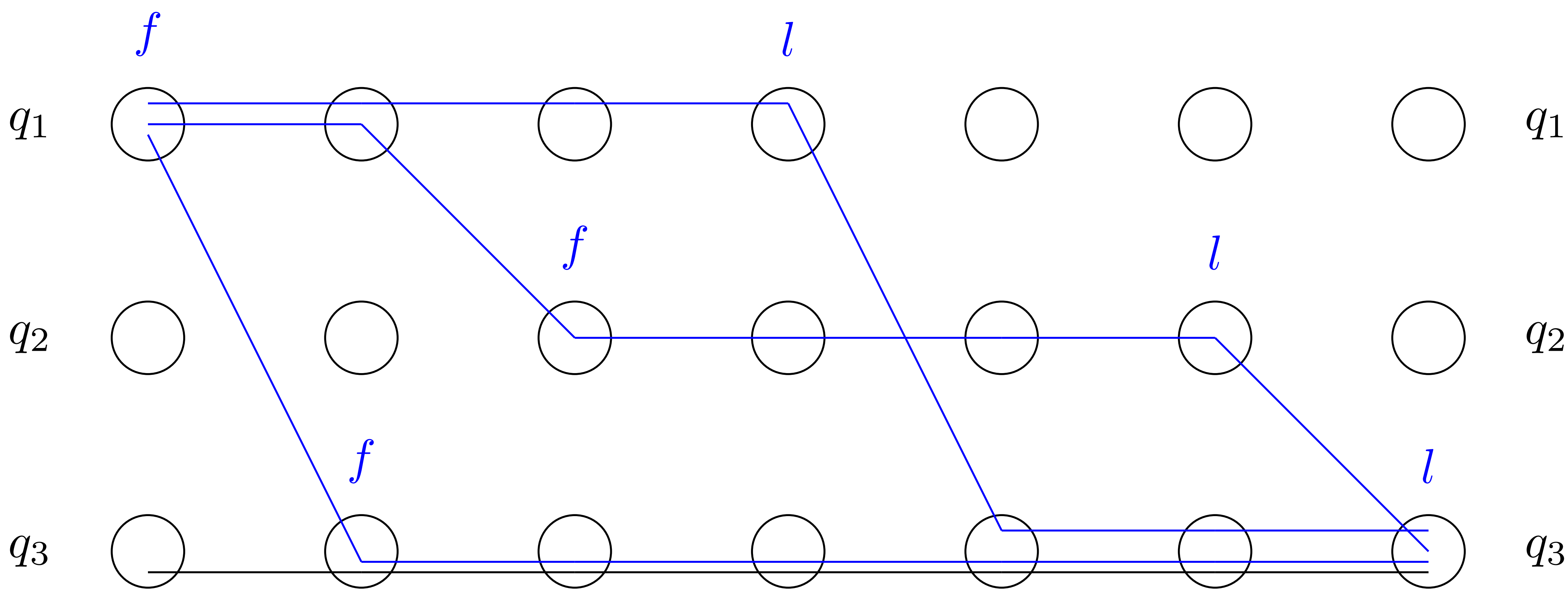

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n^2 **bunches of size** n

Bringing it together

Theorem

For *N* an IO net with *n* places, for *S* a counting set, there exists counting constraints representing *pre*(S)* and *post*(S)* whose size is bound by

$$||pre^*(S)|| \leq ||S|| + n^3$$

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contains markings with “small” number of agents

Bringing it together

CC-reachability can be solved in PSPACE.

Algorithm sketch:

Let *S* and *S'* two counting sets.

S' is reachable from *S* if and only if $S \cap pre^*(S') \neq \emptyset$

If it is non-empty, there exists a "small" marking in the intersection.

We pick such a marking in *S* and such a marking in *S'*, and then guess a path from one to the other.

Conclusion

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Thank you !

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$$post^*(\mathcal{I}) \subseteq pre^*(\mathcal{ST}_0 \cup \mathcal{ST}_1)$$

$$\wedge$$

$$pre^*(\mathcal{ST}_0) \cap pre^*(\mathcal{ST}_1) \cap \mathcal{I} = \emptyset$$

$\mathcal{I}$ the initial configurations

$\mathcal{ST}_b$ the stable b-consensus configurations for $b \in \{0,1\}$