Parameterized Analysis of Immediate Observation Petri Nets

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joint work with Javier Esparza, Mikhail Raskin

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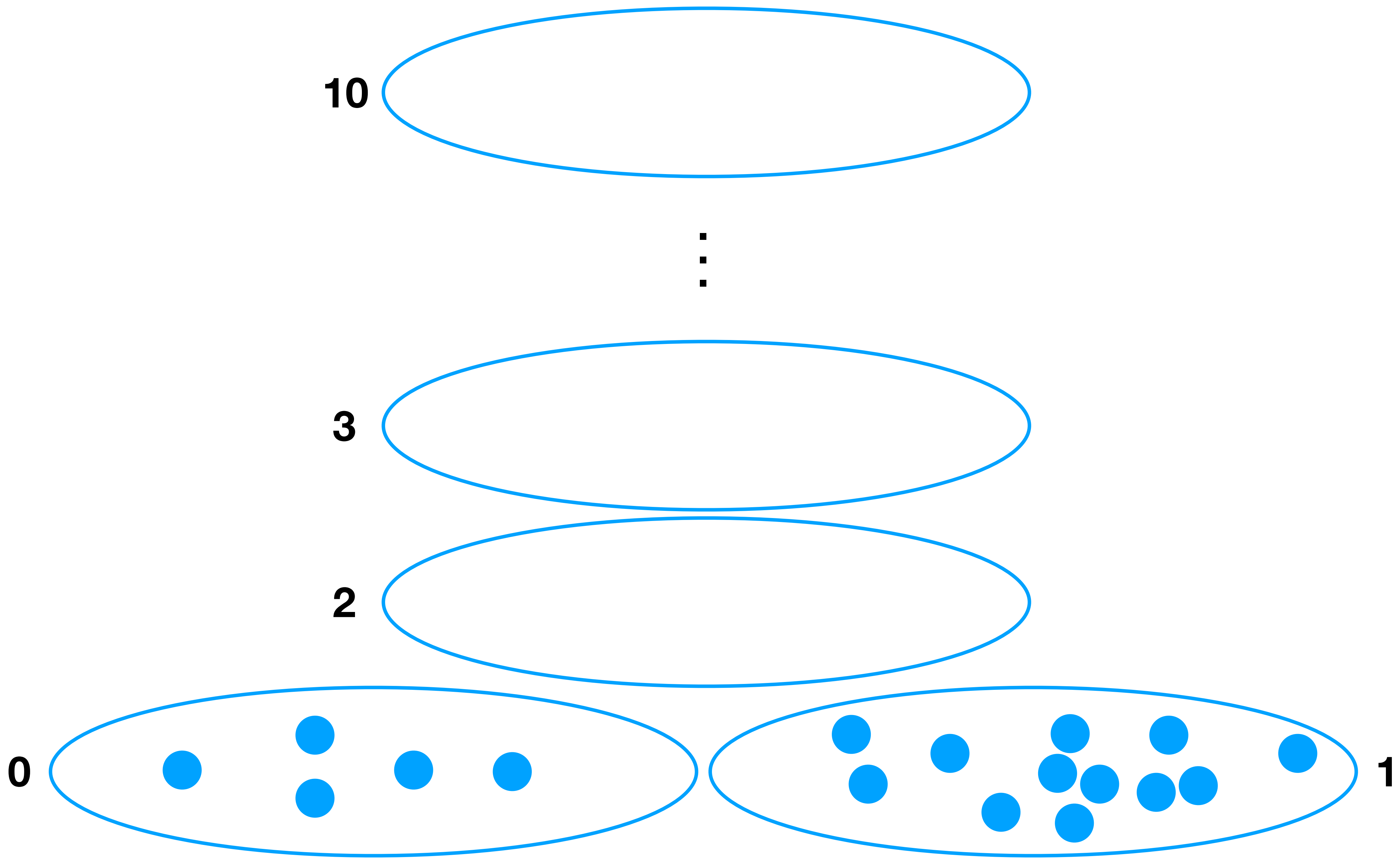
Counting Researchers Protocol



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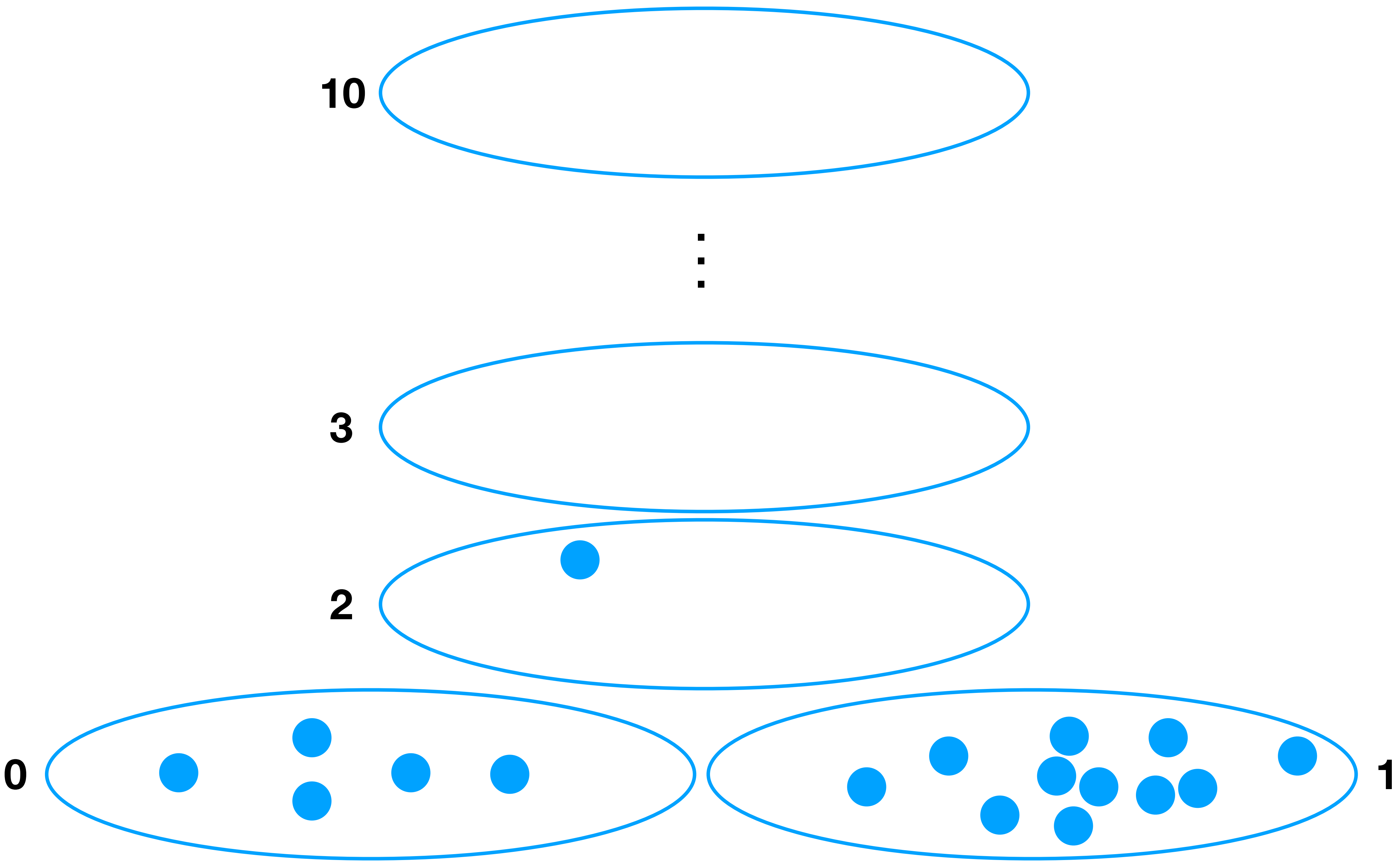

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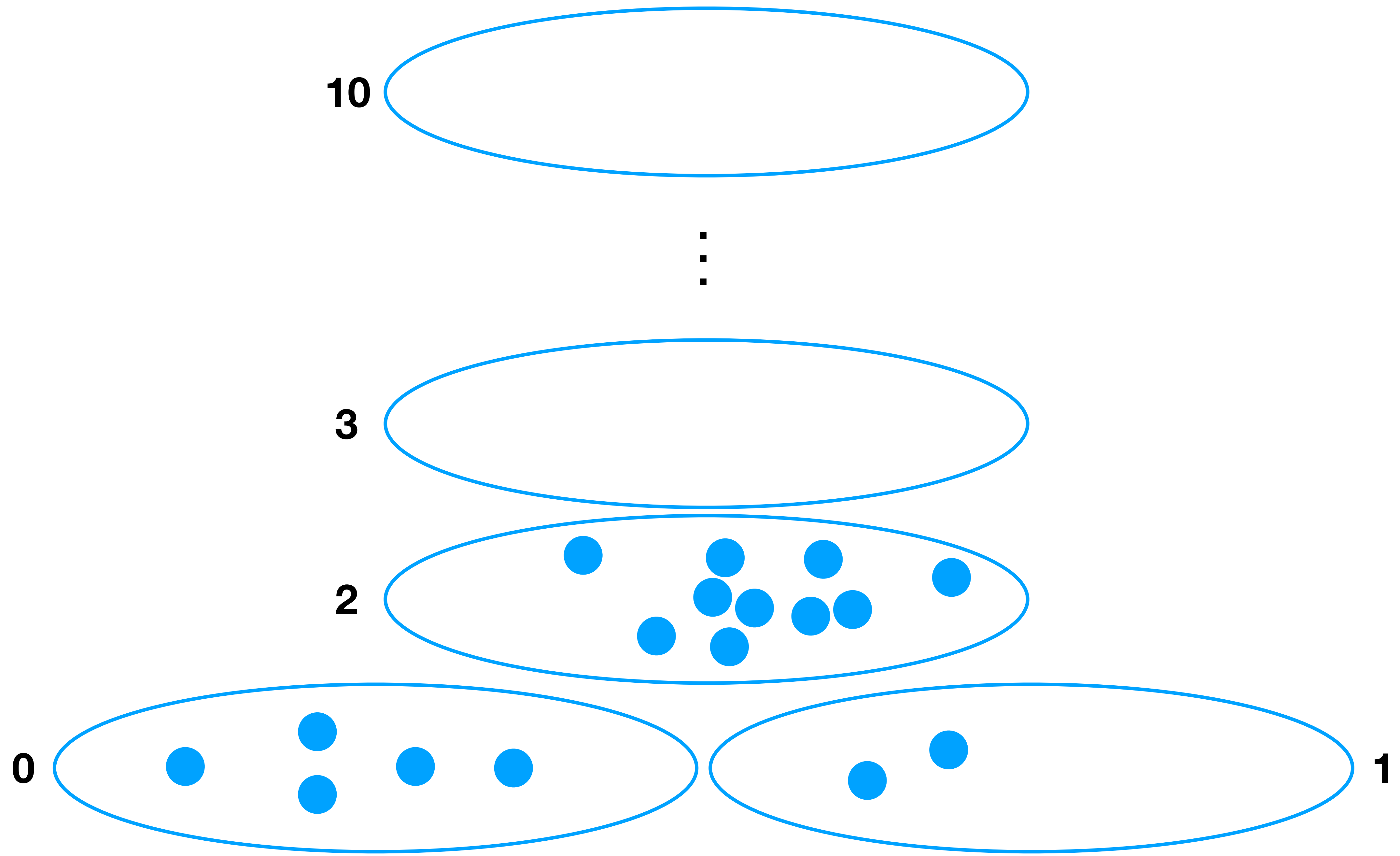

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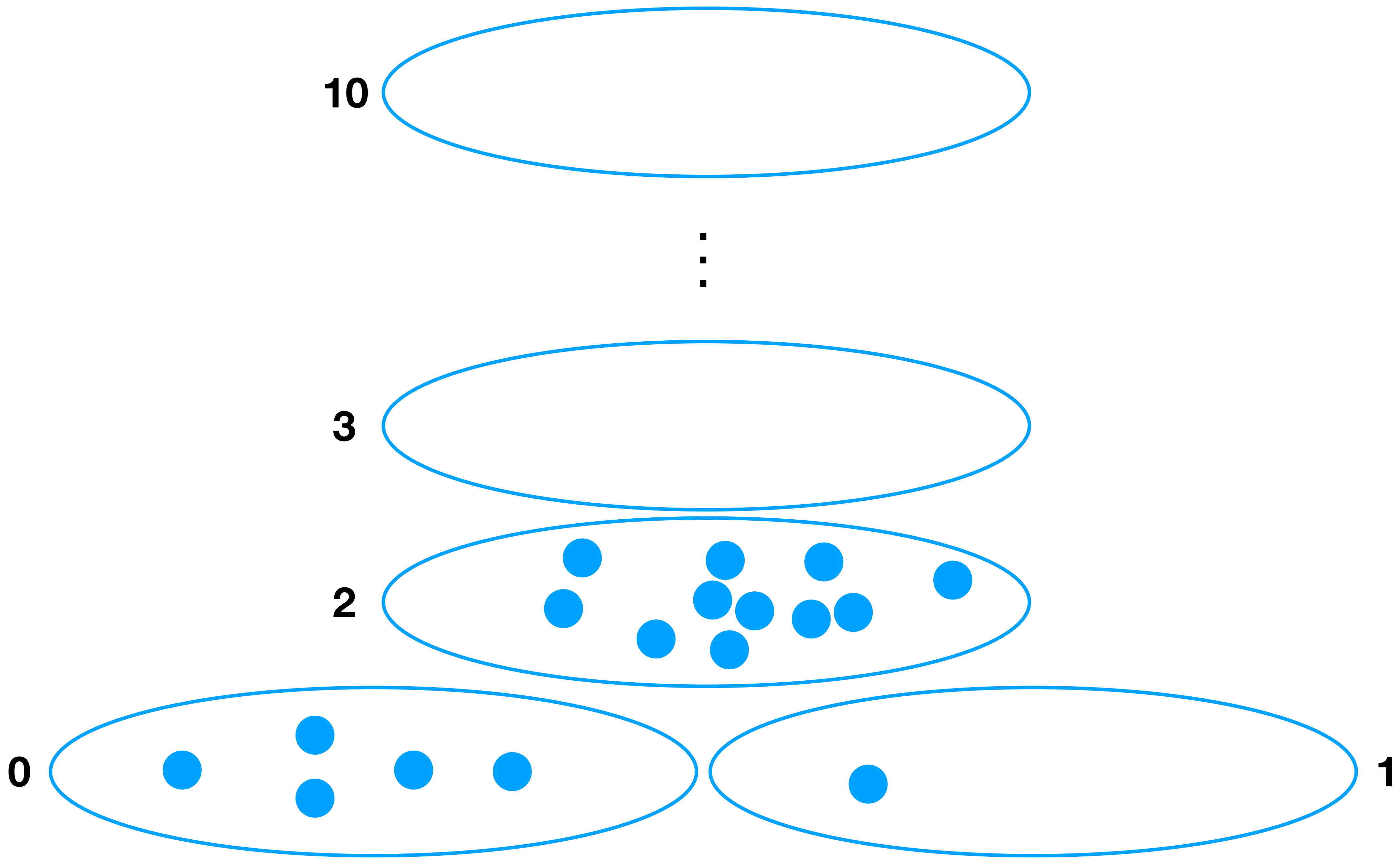

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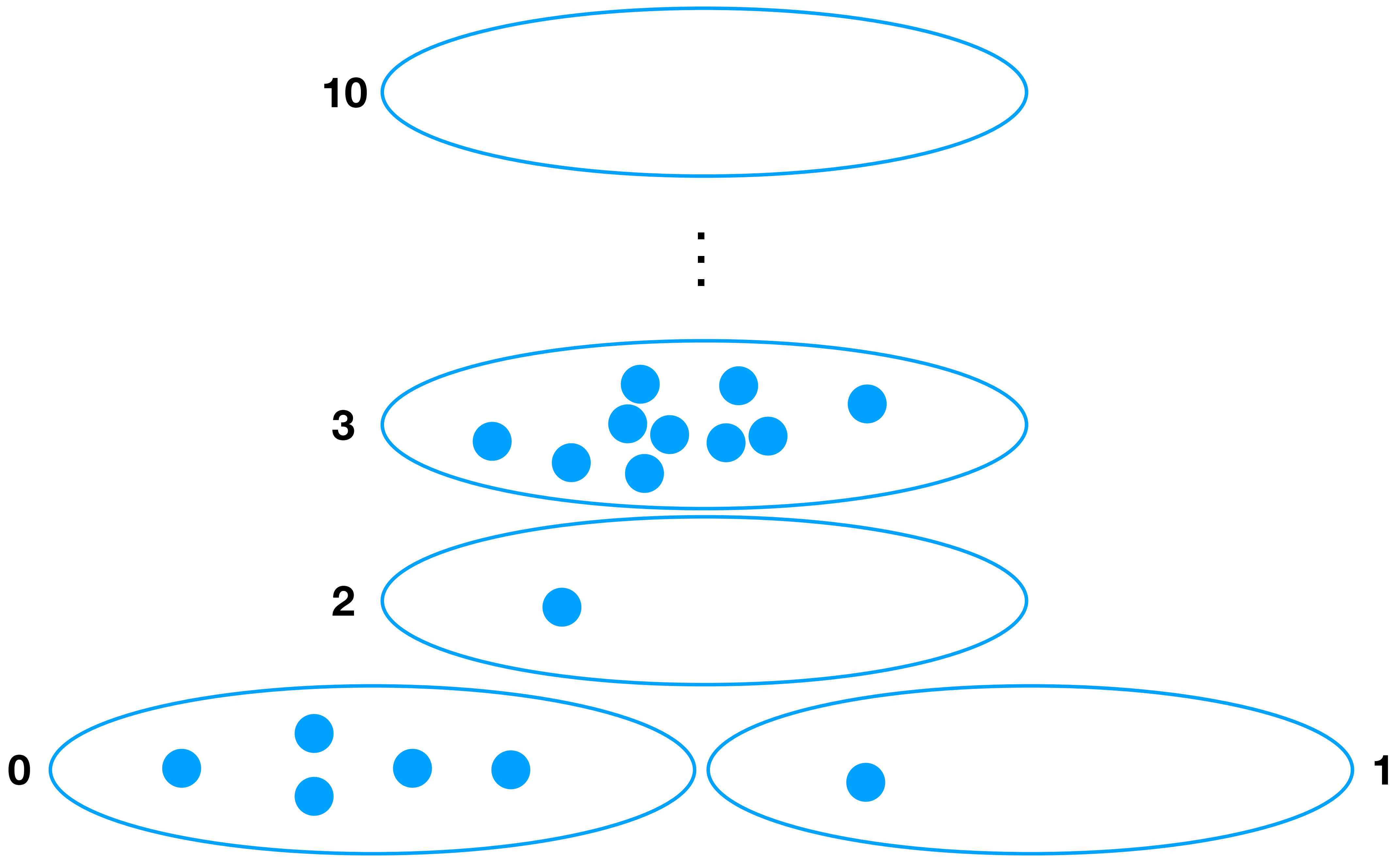

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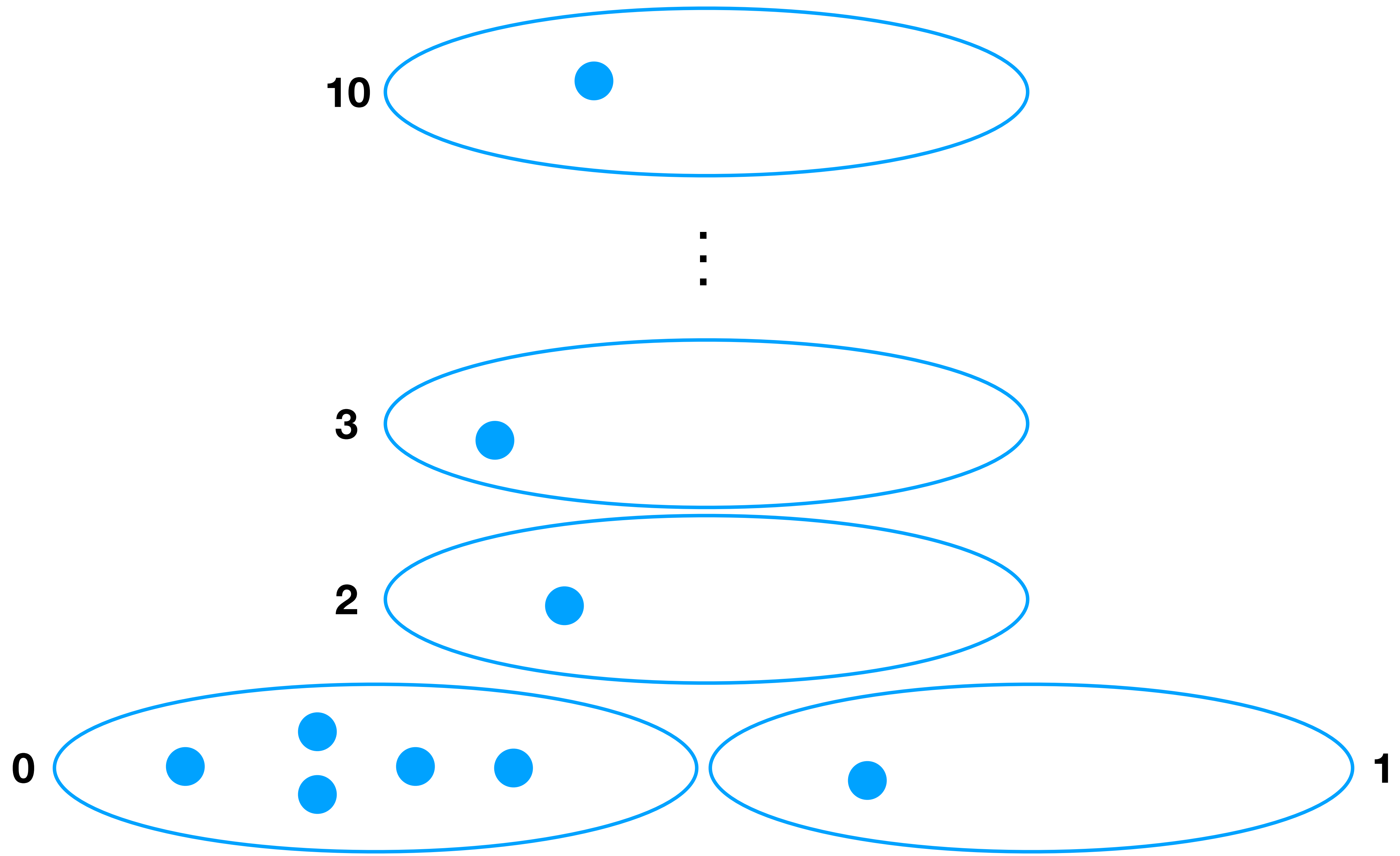

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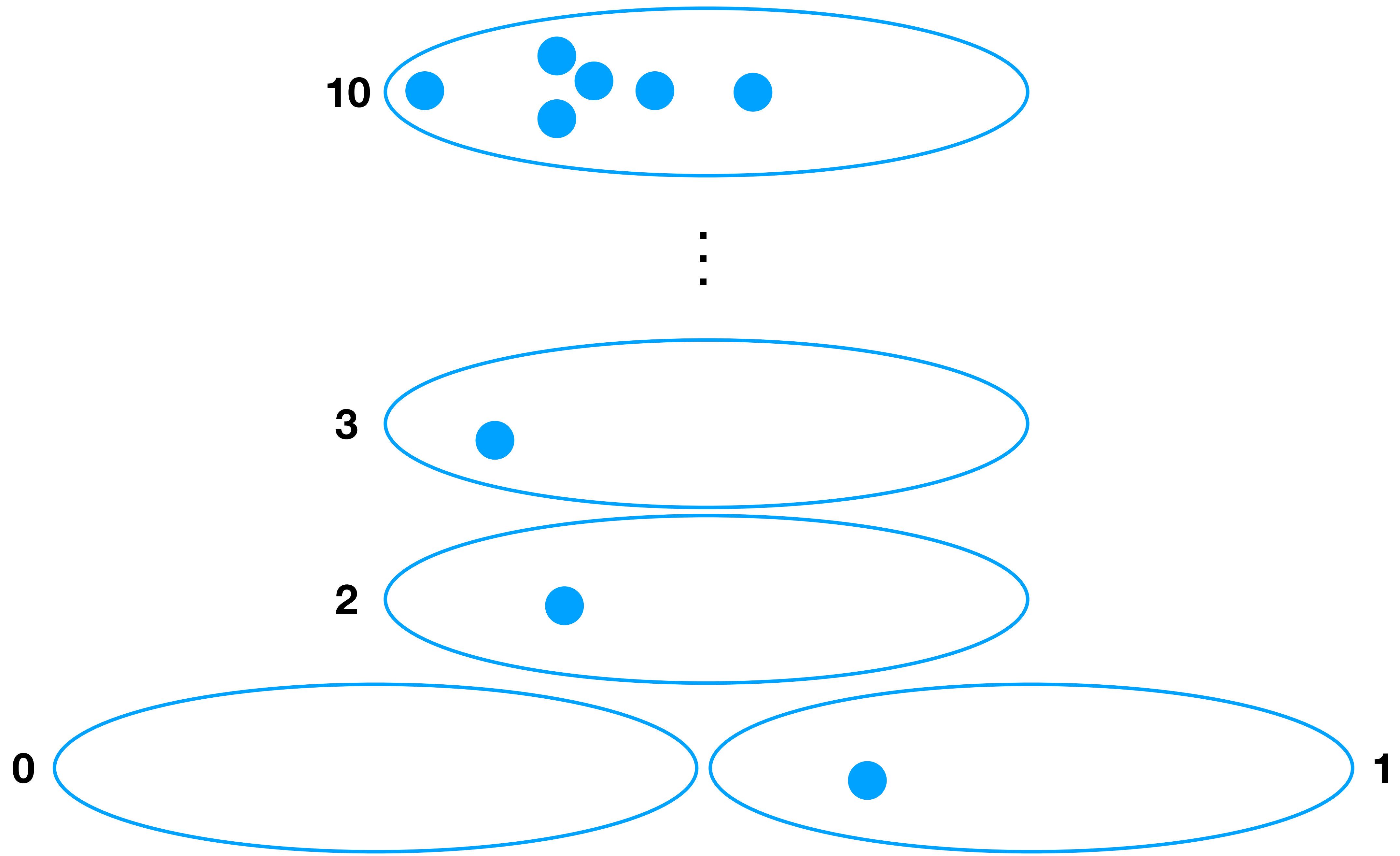

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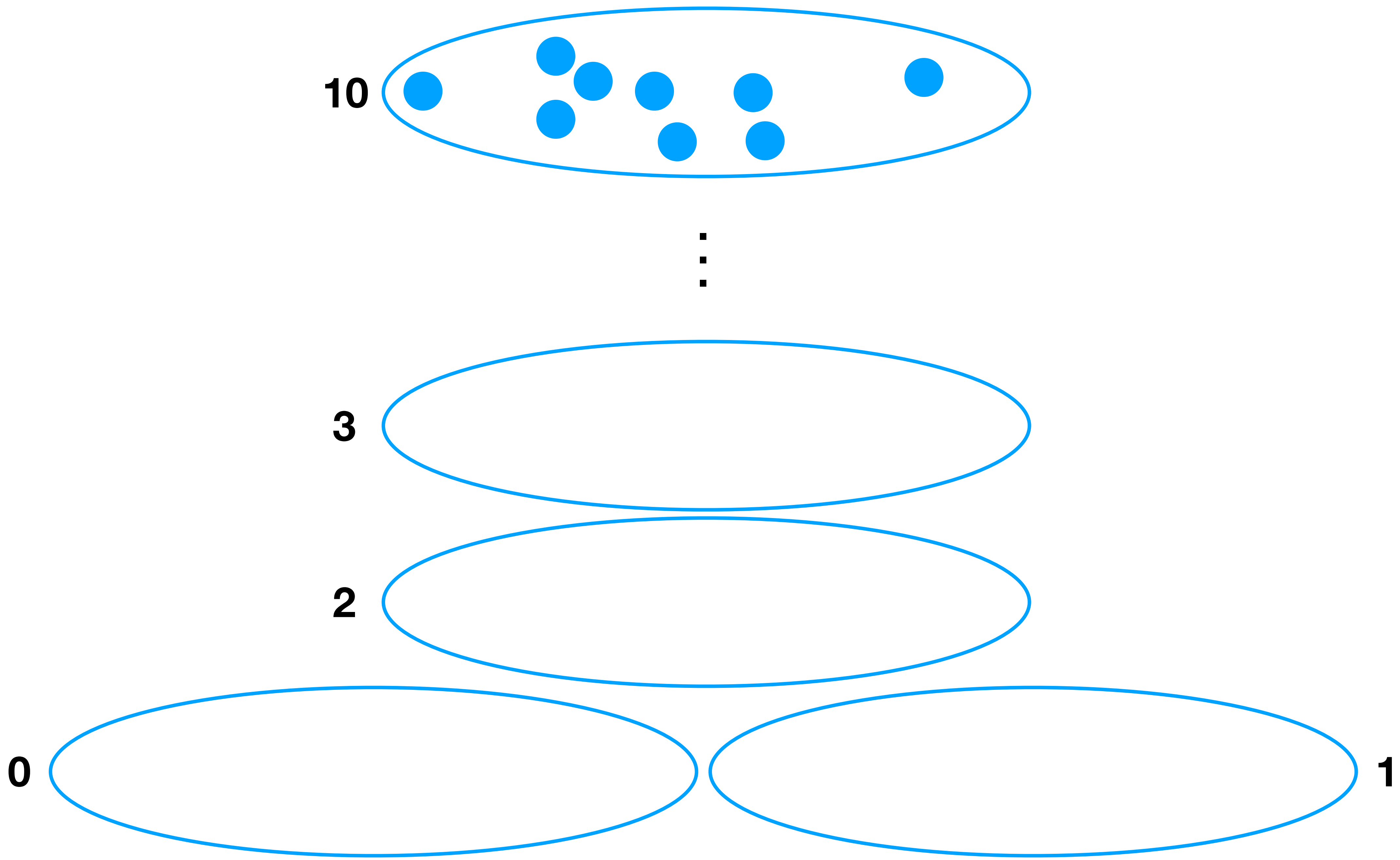
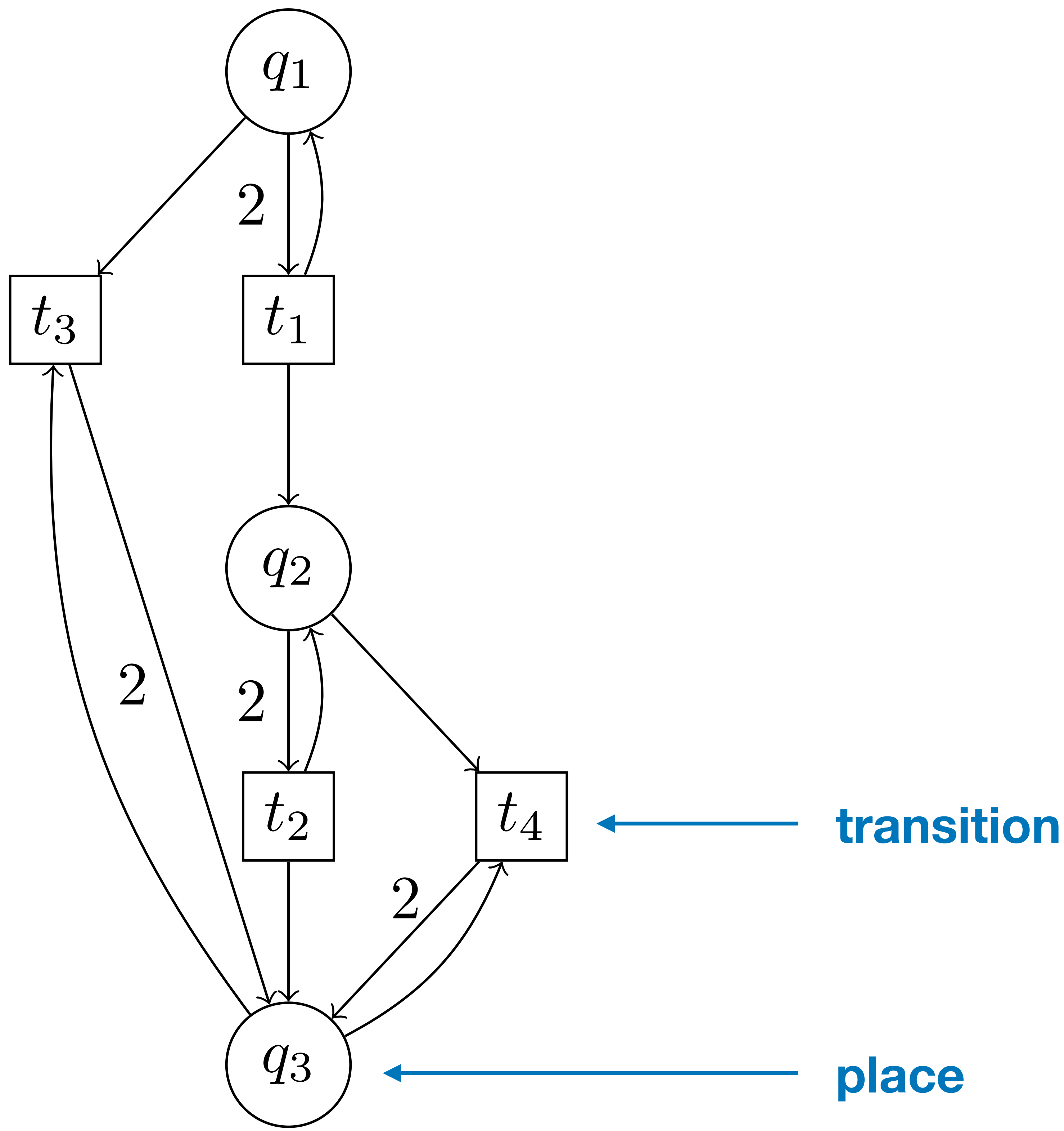


Immediate Observation Nets



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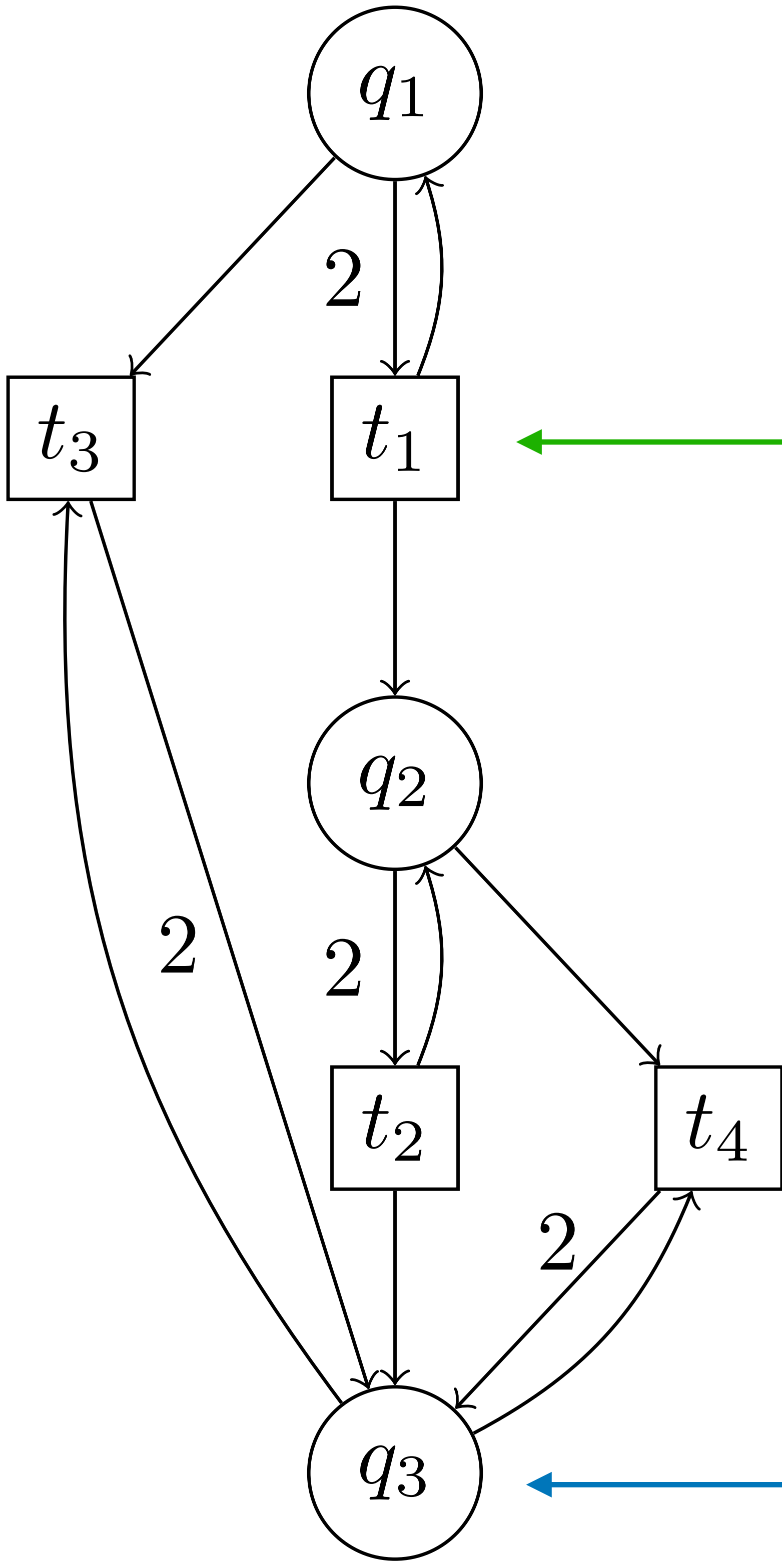

example:

$(q_1, q_1) \mapsto (q_1, q_2)$

transition

place

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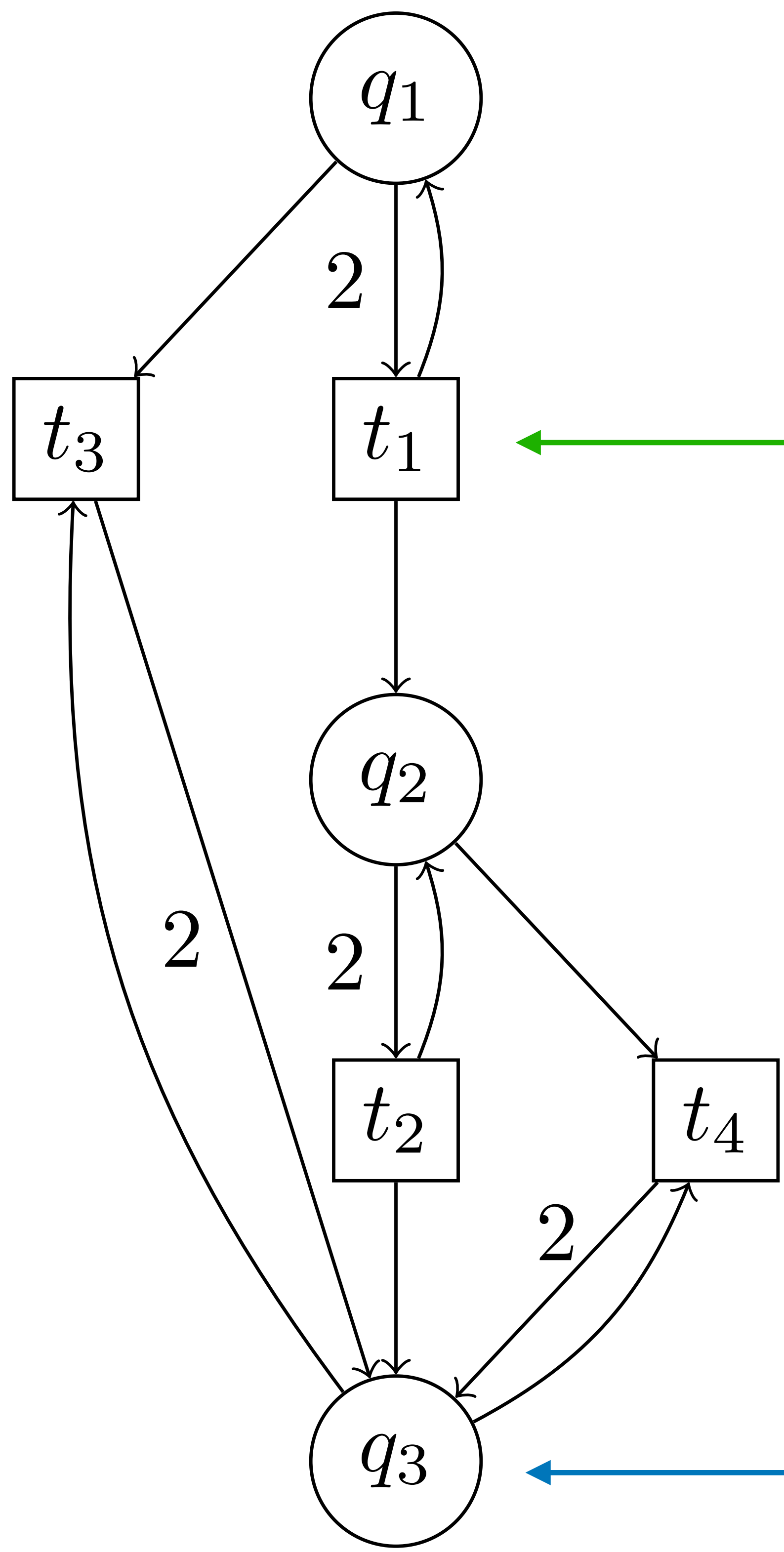

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$(q_1, q_1) \mapsto (q_1, q_2)$

A Petri net is an **IO net** if all its transitions are of the form

$(p_s\ , p_o) \mapsto (p_d\ , p_o)$

transition

place

Immediate Observation Nets

- we call **marking** a configuration of the IO net
- a **run** is a sequence of valid transitions
- each **immediate observation** population protocol has an underlying Petri net - this net is an IO net

Parameterized Problems

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We concentrate on infinite sets defined by counting constraints.

Results

- Cube-reachability, cube-coverability and cube-liveness in IO nets are **PSPACE-complete** … even though they are already PSPACE-hard for singleton sets of markings

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- Application: the correctness problem for IO population protocols is **PSPACE-complete**

How

We use

- a fundamental technique: pruning
- a useful representation: counting constraints

The Pruning Theorem

For any run

$$M \xrightarrow{*} M' \geq M''$$

there exists a run

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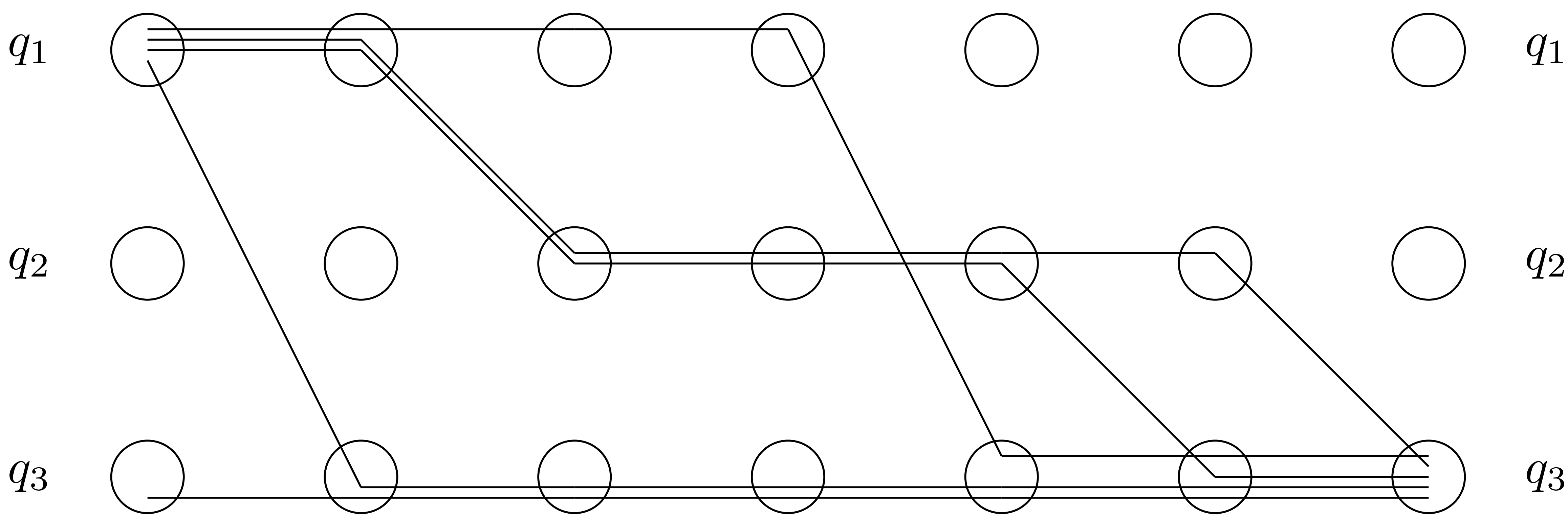

History = the trajectories of the different tokens in a run

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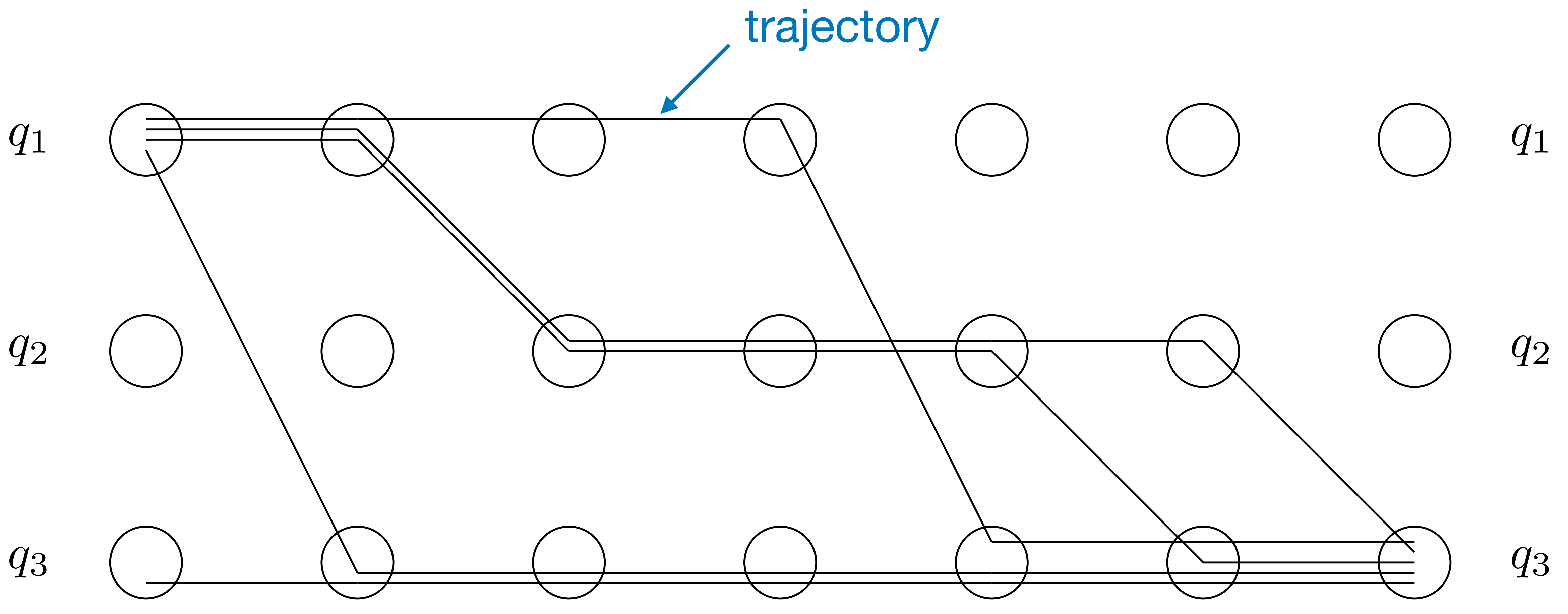

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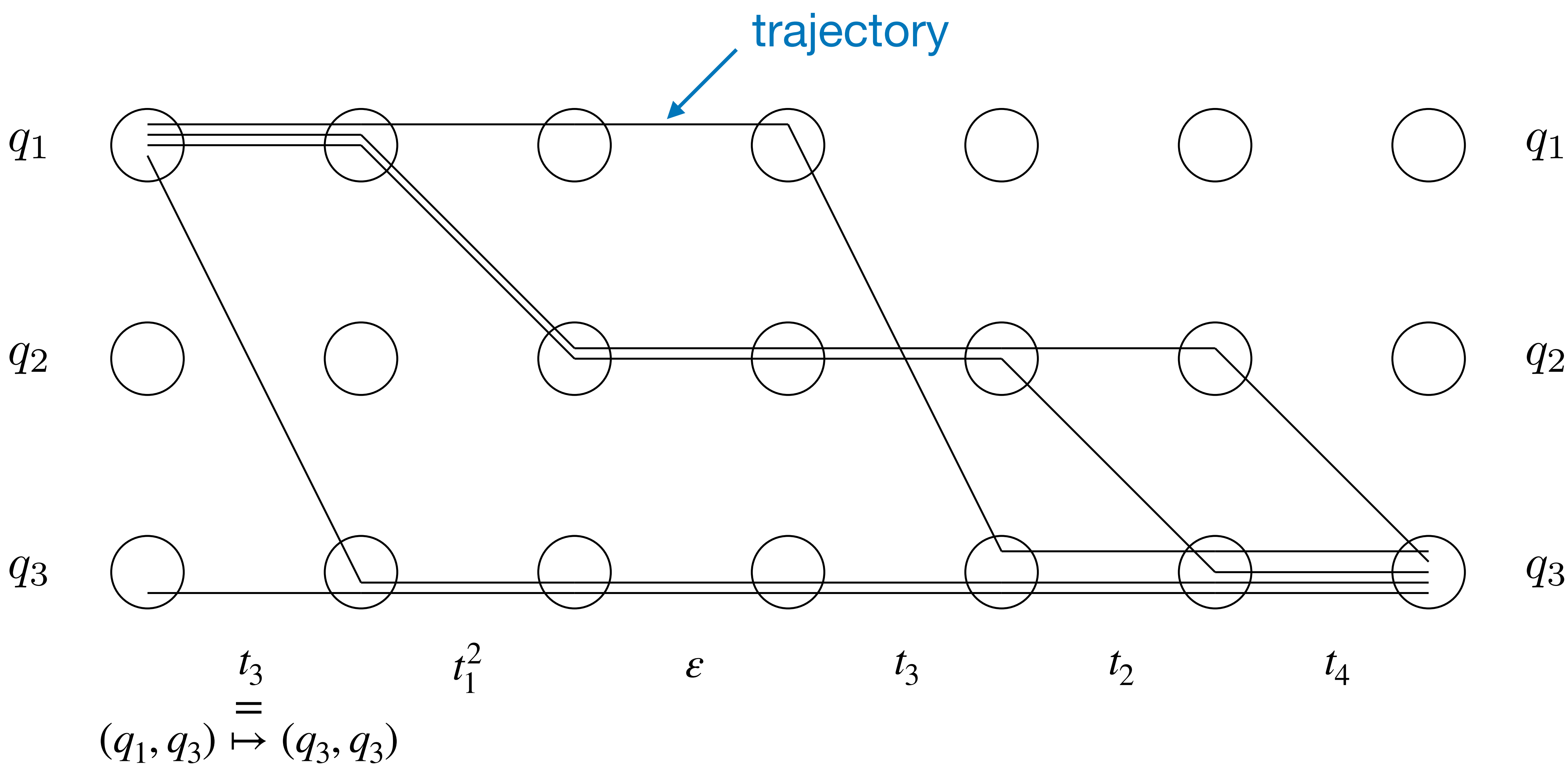

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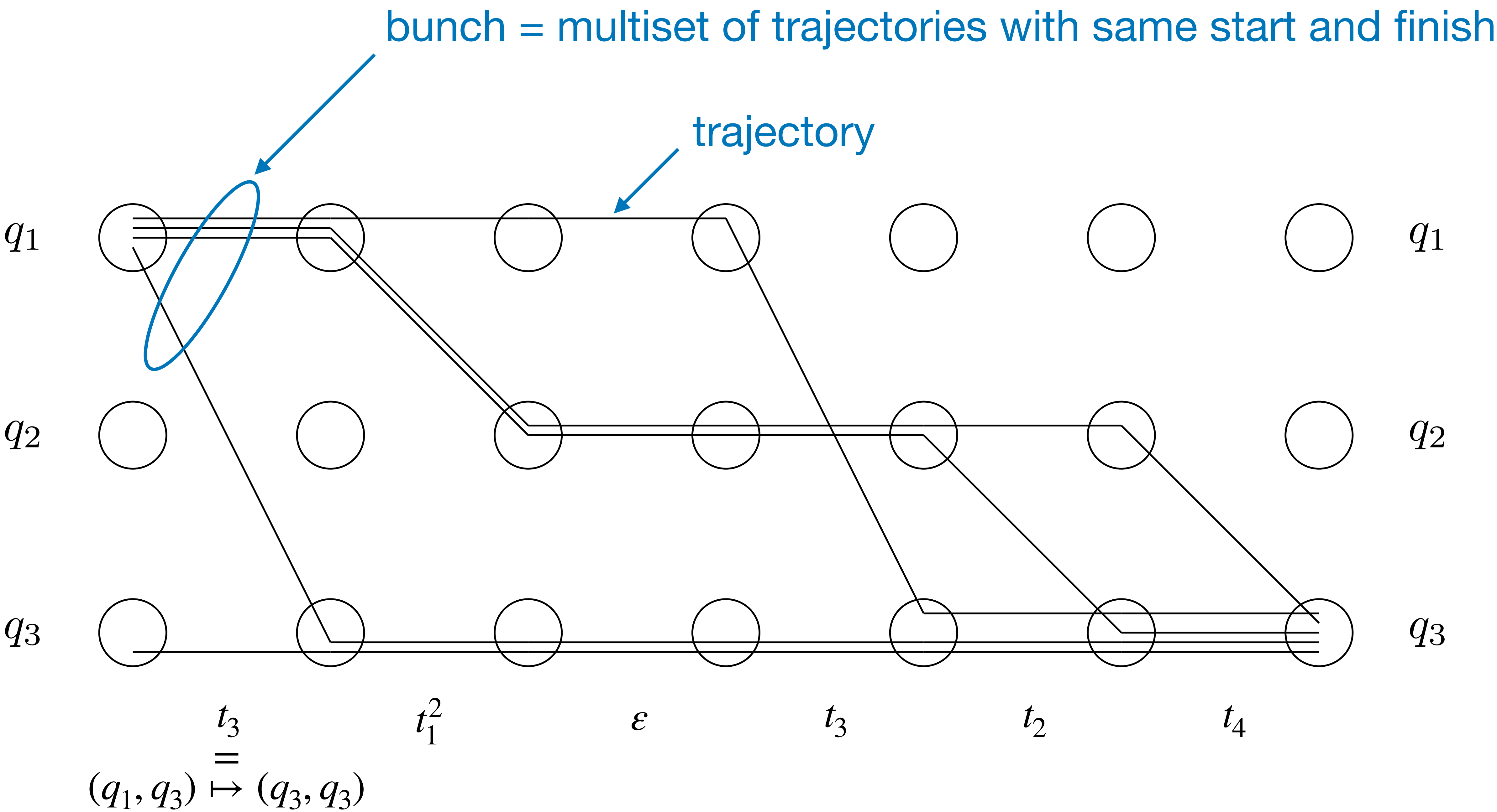

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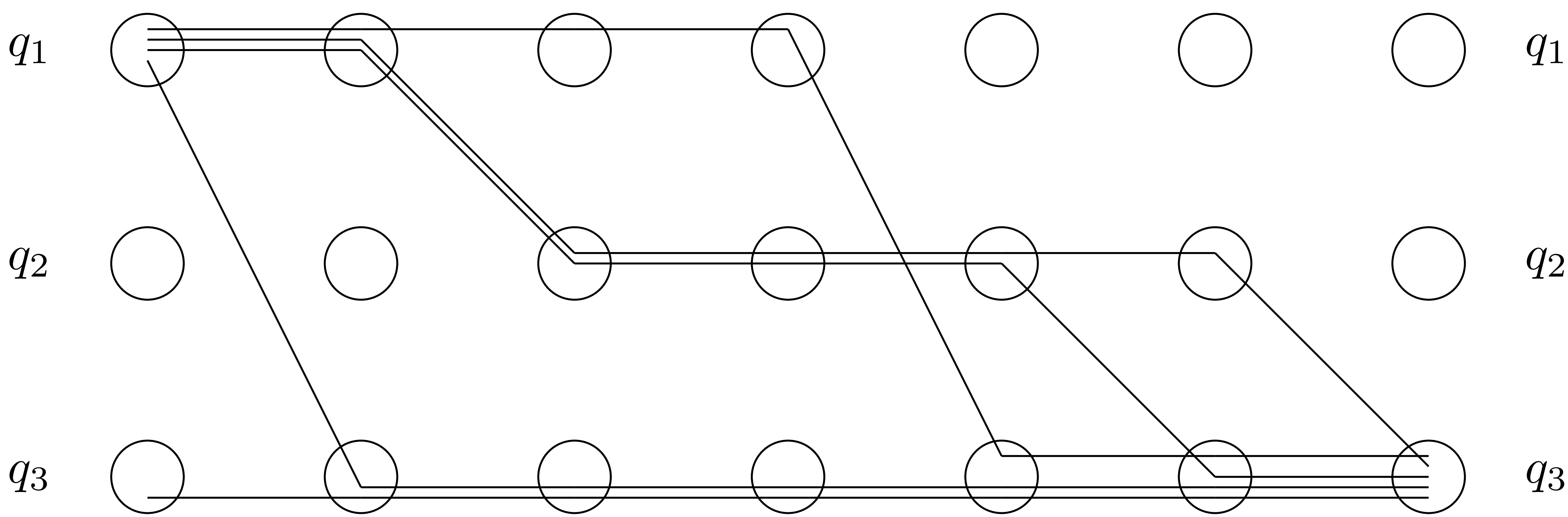
Transitions are enabled as soon as there is one token in the observed place



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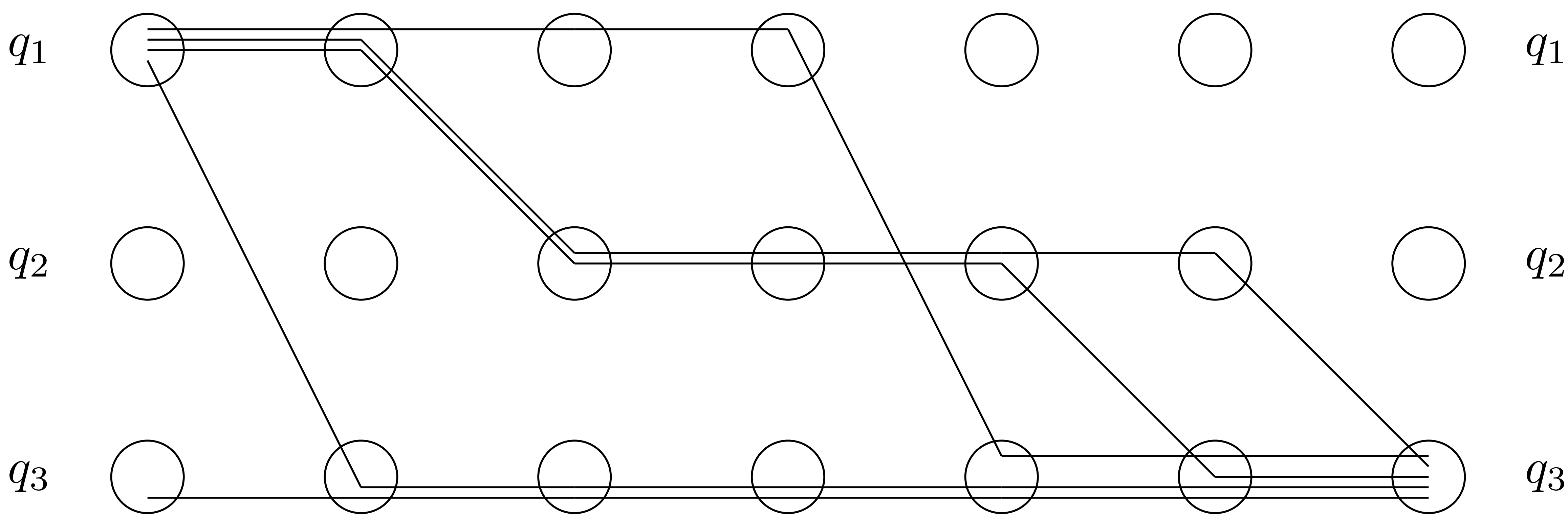

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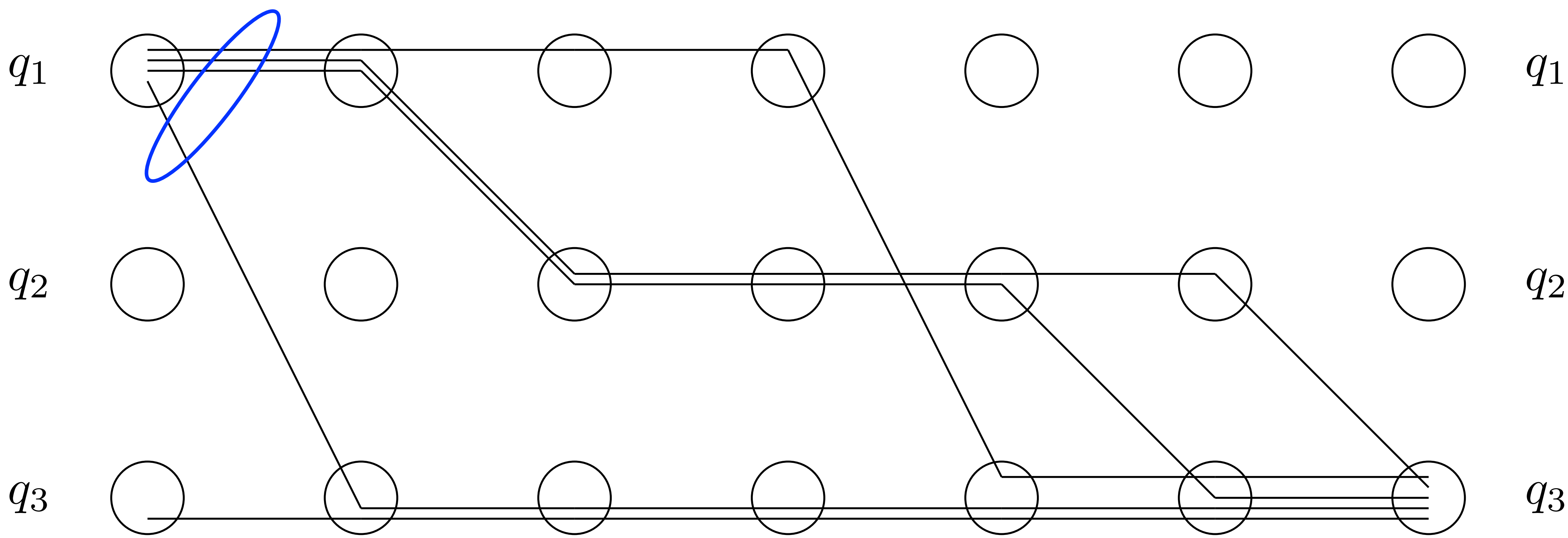

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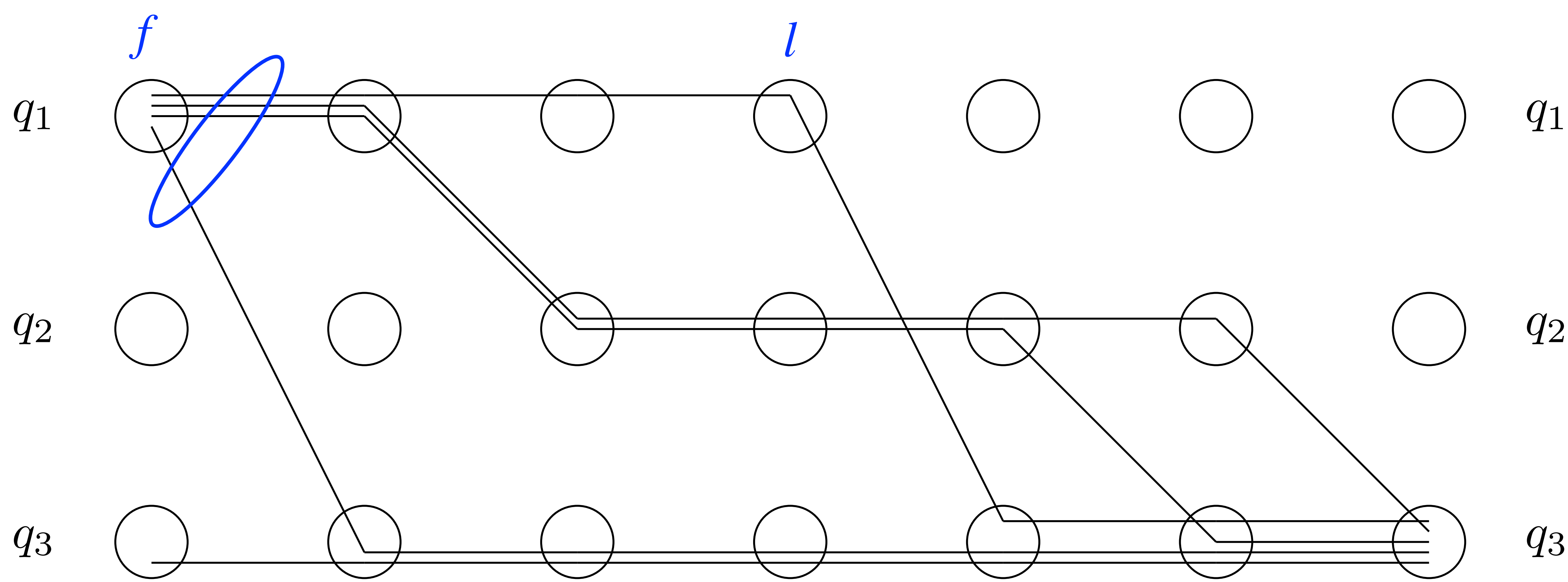

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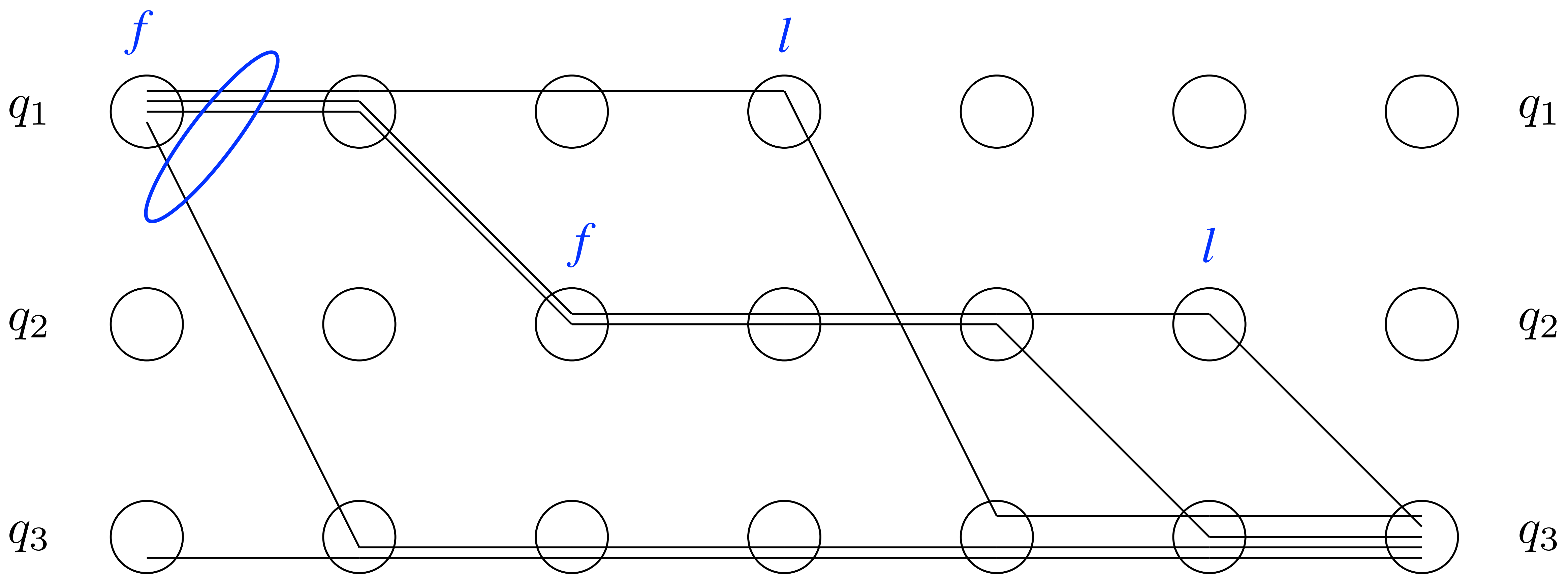

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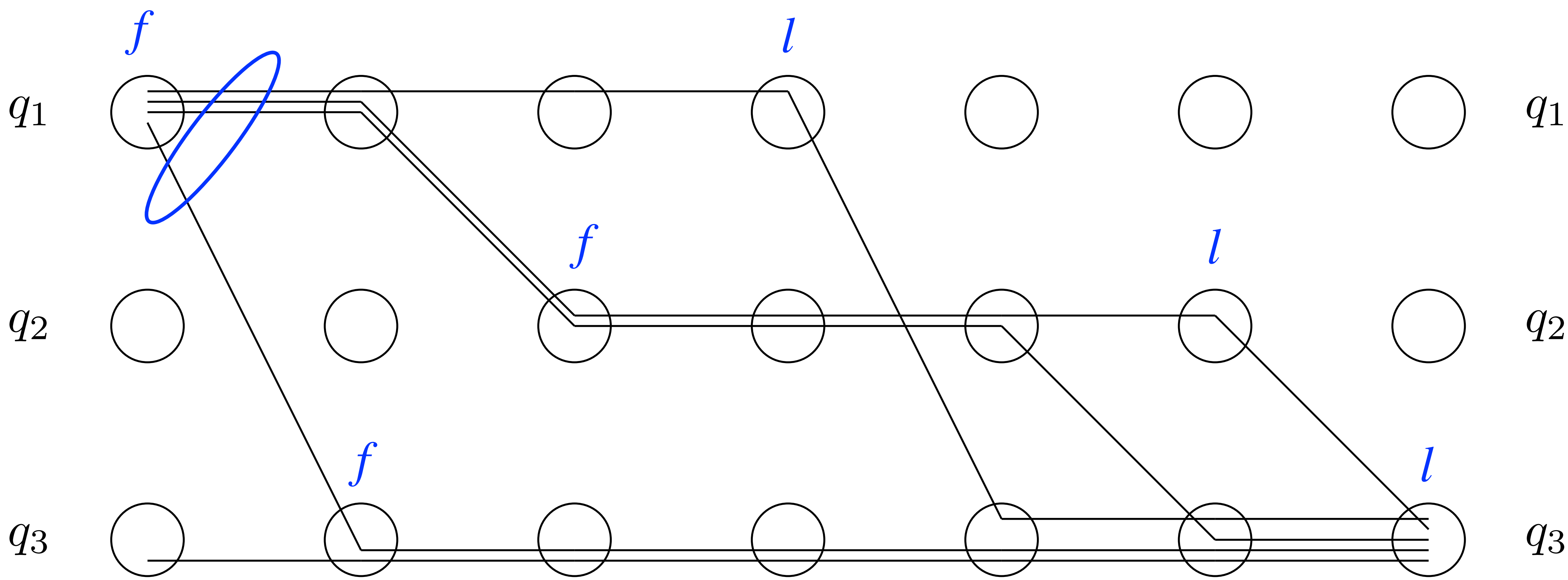

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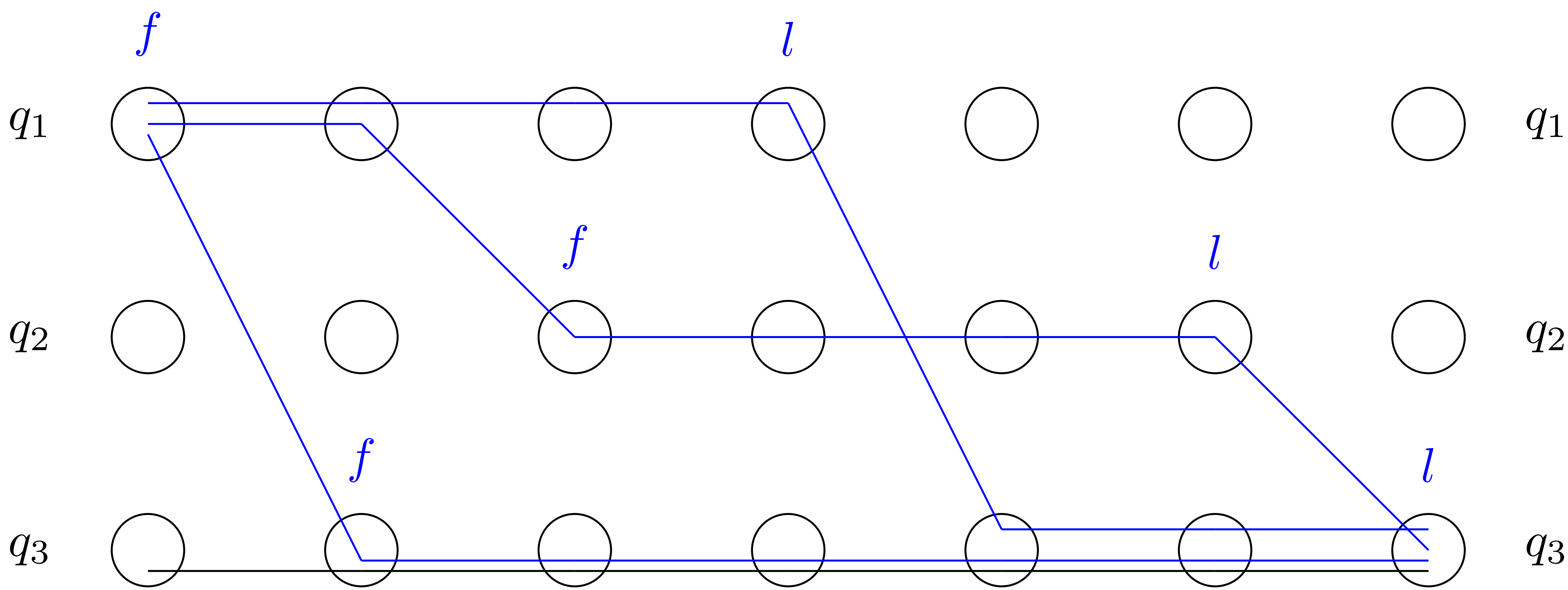

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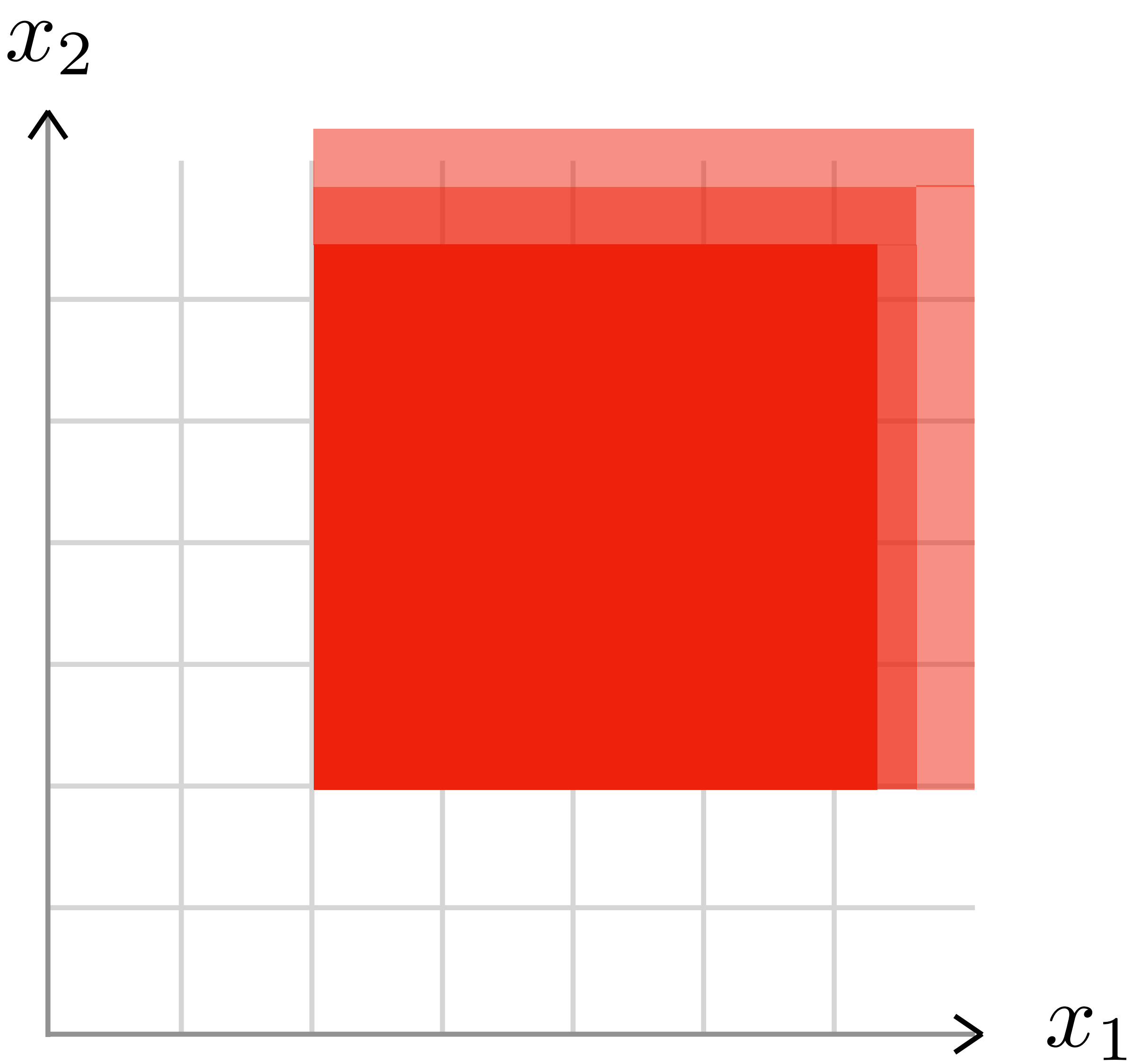
Counting Constraints



$$2 \leq x_1 \leq \infty$$
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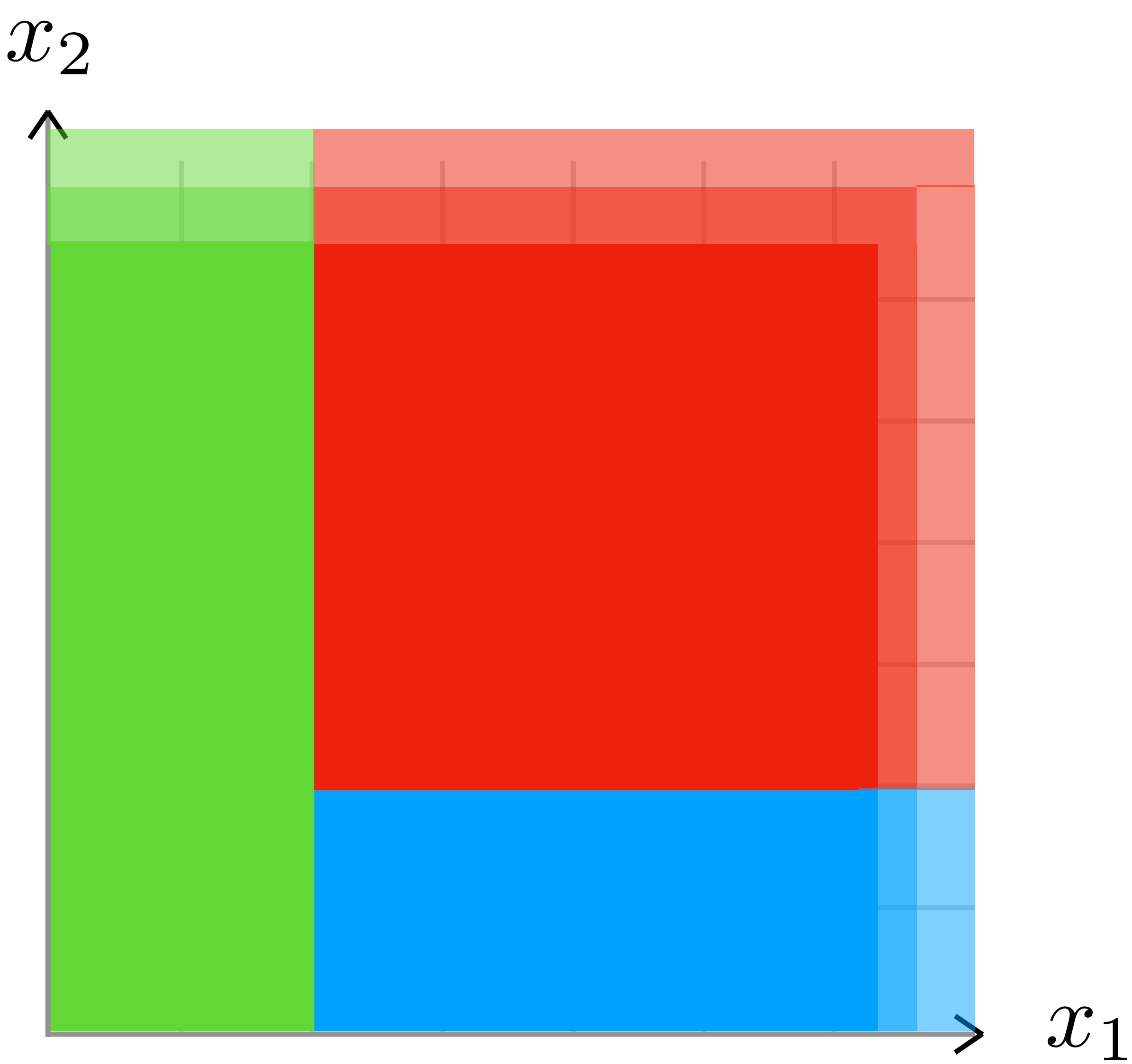

$$\neg \begin{array}{c} 2 \leq x_1 \leq \infty \\ 2 \leq x_2 \leq \infty \end{array}$$

$$\Downarrow$$

$$\begin{array}{c} 2 \leq x_1 \leq \infty \\ 0 \leq x_2 \leq 2 \end{array} \vee \begin{array}{c} 0 \leq x_1 \leq 2 \\ 0 \leq x_2 \leq \infty \end{array}$$

Bringing it together

Bound on the size of the reachability sets

For *N* an IO net with *n* places, for *S* a counting set, there exists counting constraints representing *pre*(S)* and *post*(S)* whose size is bound by

$$\|pre^*(S)\| \leq \|S\| + n^3$$

$$\|post^*(S)\| \leq \|S\| + n^3$$

Bringing it together

Cube-reachability can be solved in PSPACE

Algorithm idea: Let *S* and *S'* two counting sets.

If S' is reachable from *S,* then $S \cap pre^*(S')$ is not empty.

By the previous theorem (and other results), there exists a "small" marking in $S \cap pre^*(S')$.

We pick such a marking in *S* and such a marking in *S',* and then guess a path from one to the other.

Conclusion

- This entails the results for cube-coverability and cube-liveness in IO nets.
- The PSPACE algorithm is also induced by the cube-reachability algorithm and the fact that protocol correctness can be expressed a formula over counting sets of markings in the corresponding IO net.
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Thank you !